Game Theory

Theory of games provides mathematical model but can be useful exploring decision making in bus... as Equilibrium point of strategy.

Saddle Point, Strategy, Value of game.

Strategy
- Pure Strategy (when both the player's adopt same strategy)
- Mixed Strategy (when both the players adopt guess strategy others)

⇒ Two person Zero Sum Game.

1. Pure Strategy Game (Saddle point Exist)
2. Mixed Strategy Game (Saddle point does not Exist)

Row Minimum ⇒ ○

Column Maximum ⇒ □

Saddle Point ⇒ ⊡

Q.

Firm B

Firm A	B_1	B_2	B_3	B_4	B_5
A_1	3	(−1)	4	6	7
A_2	(−1)	8	2	4	[12]
A_3	[16]	8	[(6)]	[14]	[12]
A_4	1	[11]	(−4)	2	1

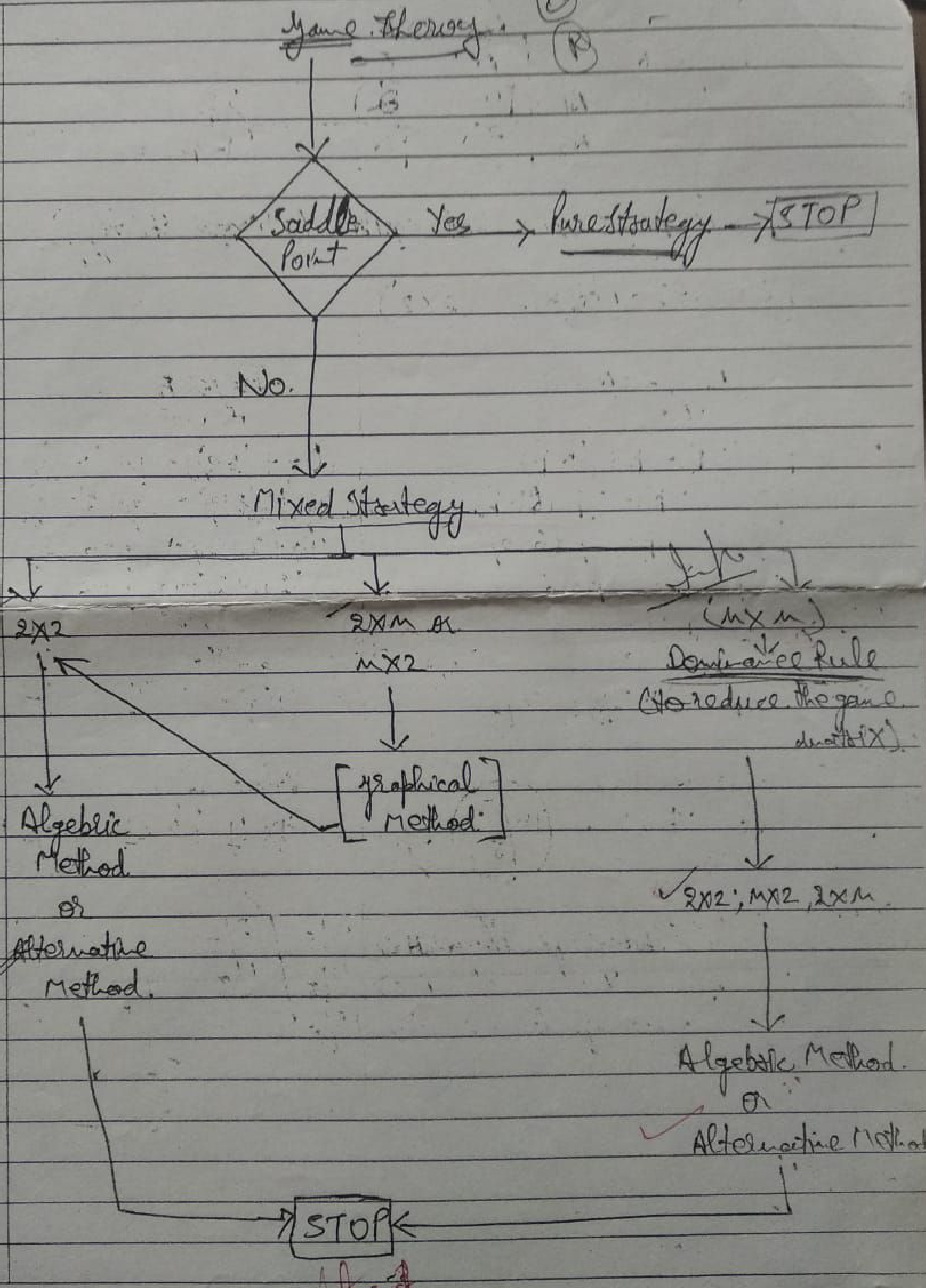
pay off
profit / loss

Optimal Strategy for Firm A = A_3 = 6

" " " Firm B = B_3 = 6

Value of game (V) = 6

Kreator


Game Theory
Saddle Point
Yes
Pure Strategy
STOP
No.
Mixed Strategy
2X2
2XM or MX2
(MXM)
Dominance Rule
(to reduce the game matrix)
Graphical Method
Algebric Method
or
Alternative Method.
2X2, MX2, 2XM
Algebric Method
or
Alternative Method
STOP

DATE
PAGE No.

2×2

$q \quad (1-q)$

Player B

Player A	B_1	B_2	
A_1	4	2	p
A_2	3	6	$(1-p)$

Saddle point does not exist (Mixed Stategy)

then use the

Alternative Method. (2×2)

Player A

let 1=1

$p+(1-p) = p+(1-p)$

$4p+3(1-p) = 2p+6(1-p)$

$4p+3-3p = 2p+6-6p$

$p+3 = -4p+6$

$5p = 3$

$p = 3/5$

$(1-p) = 1-3/5 = 2/5$

Value of game.

$4\times\frac{3}{5}+3\times\frac{2}{5} = 18/5$

Player B

1=1

$q+(1-q) = q+(1-q)$

$4q+2(1-q) = 3q+6(1-q)$

$4q+2-2q = 3q+6-6q$

$2q+2 = -3q+6$

$5q = 4$

$q = 4/5$

$(1-q) = 1-4/5 = 1/5$

Value of game,

$4\times\frac{4}{5}+2\times\frac{1}{5} = 18/5$

Opttimal Stategy for Player A ⇒ 3/5 | 2/5

" " " B ⇒ 4/5 | 1/5

Value of Game ⇒ 18/5

DATE
PAGE No.

2×n

Graphical Method → used to convert the matrix to 2×2.

Ⓐ Ⓒ

① 2×n = (maximin → Maximum out of Minimum)

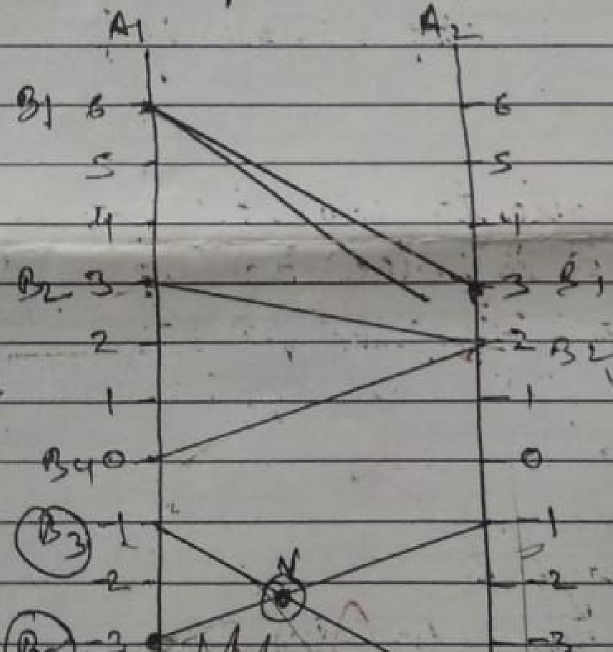
② n×2 = (minimax → Minimum out of Maximum)

R×C 2×n

Player B

Player A	B_1	B_2	B_3	B_4	B_5
A_1	6	3	−1	0	−3
A_2	3	2	−4	2	−1

Saddle point does not Exist. (Mixed Strategy)



		2/5	3/5
		B_3	B_5
3/5	A_1	−1	−3
2/5	A_2	−4	−1

Value of game = $-11/5$

- Optimal Strategy for Player A – 3/5, 2/5

Optimal Strategy for Player B – (0, 0, 2/5, 0, 3/5)

Value of game = $-11/5$

(2×2)

Dominance Rule.

Rule ① ⇒ Compare any two rows & remove the row which is smaller or equal from other.

Rule ② ⇒ Compare any two column & remove the columns which is greater or equal to other.

Rule ③ ⇒ If we are unable to reduce the matrix by rule ① or rule ②, then we can reduce by using this rule. compare the rows with average of other rows & remove if smaller or equal.

Rule ④ ⇒ If we are unable to reduce the matrix by rule ① & rule ② or rule ③ then we can reduce by using this rule compare the column with average of other column & remove if greater or equal.

N×N

Ex.

Player B.

Player A	B_1	B_2	B_3	B_4			
A_1	3	2	4	0			
A_2	3	4	2	4		6	10
A_3	4	2	4	0		4	8
A_4	0	4	0	8		8	12
		10	6	12			

No, Saddle point does not Exist (Mixed Strategy)

Row $A_1 \leq A_3$ so remove A_1, make a new matrix

	B_1	B_2	B_3	B_4
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

Column $B_1 \geq B_3$ so remove B_1

	B_2	B_3	B_4
A_2	4	2	4
A_3	2	4	0
A_4	4	0	8

Column $B_2 \geq$ Avg. (B_3, B_4) so remove B_2

	B_3	B_4
A_2	2	4
A_3	4	0
A_4	0	8

Row $A_2 \leq$ Avg. $(A_3 \& A_4)$ so remove A_2

	B_3 (q)	B_4 $(1-q)$	
A_3	4	0	p
A_4	0	8	$(1-p)$

$p = 2/3$, $(1-p) = 1/3$ — Value of game $= 8/3$

$q = 2/3$, $(1-q) = 1/3$ — Value of game $= 8/3$

Optimal strategy for Player A $= (0, 0, 2/3, 1/3)$

" " B $= (0, 0, 2/3, 1/3)$

Value of game $= 8/3$