Unit → 5

Test of variance →

(1) **Chi - square test** → It is used to test whether the difference b/w the observe and expected frequency can be calculated by chance ~~for~~ or not.

If the difference b/w the expective and the observe frequency is zero it means the result is ~~expe~~ accepted otherwise rejected.

Procedure for chi - square test → It is denoted by χ^2 or ψ^2

Step(1) → Null hypothesis

Step(2) → Calculation of chi - square

χ^2

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(f_o - f_e)^2}{f_e} \right] = \sum_{i=1}^{n} \left[\frac{(O-E)^2}{E} \right]$$

where

$O = f_o$ = is the observed frequency

$E = f_e$ = is the expected frequency

Step (3) → Test the hypothesis

If $\chi^2 \leq$ table value it means result is accepted.

If $\chi^2 >$ table value it means result is rejected.

Q → The following table gives the no of accident that took place in an industry during various days of the week. Test of accidents are uniformly distributed over the week.

Day	mon	Tue	wed	Thu	fri	Sat
No of accidents	14	18	12	11	15	14

Table value of x^2 at 5% level is 11.09.

Step (1) → Null hypothesis

$$H_0 = \frac{\text{Total no of accident}}{n} = \frac{84}{6} = 14$$

$H_0 = 14$

Step(2) → Calculation of chi - square

$$\chi^2 = \sum_{i=1}^{n}\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

f_o	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$
14	14	0	0
18	14	4	16
12	14	-2	4
11	14	-3	9
15	14	1	1
14	14	0	0
	84		30

$$\frac{30}{84} = 0.357$$

Step(3) → Test the hypothesis

$\chi^2 = 0.356$

Table ~~Total~~ value of $\chi^2 = 11.09$

$\chi^2 <$ table value

So, the sol^n is accepted

Q → A dice is thrown 270 times & the result of these throws are given below -

No. appeared on the dice	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the dice is biased or not.

Step(1) → Null hypothesis

$H_0 = \frac{270}{6} = 45$

Step(2) → Calculation of chi-square

$$\chi^2 = \sum_{i=1}^{n}\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$
40	45	-5	25
32	45	-13	169
29	45	-16	256
59	45	14	196
57	45	12	144
59	45	14	196
	270		986

Step (3) → Test the hypothesis

$\chi^2 = \underline{3.65}$

Table value of $\chi^2 = \underline{11.09}$

$\chi^2 <$ table value

So, the sol^n is accepted.

Q → Record ~~tabt~~ taken of the no. of male & female births in $\underline{800}$ families having $\underline{4}$ children are as method -

No. of male birth	0 (n)	1	2	3	4
No. of female birth	4	3	2	1	0
No. of families	32	178	260	236	94

Test whether the data are consistent with hypothesis that the binomial law holds & the chance of male birth is equal to that of female birth namely $\underline{p = q = 1/2}$.

[Tabulated value of $\chi^2 = 9.49$]

Step (1) → Null hypothesis — total no. of children.

$p = q = 1/2$

~~Step (2)~~ → Calculate the expected frequency

Let r be the no. of male in a family. $n = 4$

$N(r) = N\, {}^nC_r p^r q^{n-r}$ $\qquad N\, {}^nC_r p^r q^{n-r}$

$N(0) = 800 \times {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4$ $\qquad r = 0$

$= 800 \times \frac{1}{16}$

$= \underline{50}$ $\qquad r = 1$

$N(1) = 800 \times {}^4C_1 p^1 q^3$ $\qquad r = 2$

$= 800 \times \frac{4!}{3!1!} \times \frac{1}{2} \times \frac{1}{8}$ $\qquad r = 3$

$= \underline{200}$ $\qquad r = 4$

$N(2) = 800 \times {}^4C_2 p^2 q^2$

$= \underline{300}$

$N(3) = 800 \times {}^4C_3 p^3 q^1$

$= \underline{200}$

$N(4) = 800 \times {}^4C_4 p^4 q^0$

$= \underline{50}$

Step (2) → Calculation of chi^2

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

f_o	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$
32	50	-18	324
178	200	-22	484
260	300	-40	1600
236	200	36	1296
94	50	44	1936
	800		5640

$$\chi^2 = \frac{5640}{800} = 7.05$$

Step (3) → Test the hypothesis

$\chi^2 = 7.05$

Table value of $\chi^2 = 9.49$

$\chi^2 <$ table value

So, the sol^n is accepted.

(2) T test → Case (1) → when only one sample is given -

The t-test is defined as

$$t = \frac{\bar{x} - \mu}{S} \times \sqrt{n}$$

where:-

* $\bar{x}$ is the mean
* μ is the mean of population
* n is the size of sample
* S is the S.D.

$$S = \sqrt{\frac{(x-\bar{x})^2}{n-1}}$$

Case (2) → when two independent samples are given -

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{S} \times \sqrt{\frac{n_1 \times n_2}{n_1 + n_2}}$$

where:-

* $\bar{x}_1$ → mean of 1st sample
* $\bar{x}_2$ → mean of 2nd sample

$$S = \sqrt{\frac{\Sigma(x_1-\bar{x}_1)^2+\Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}}$$

If two samples S_1 & S_2 are given -

$$\text{the } S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1+n_2-2}$$

Q→ Two samples of sodium vapor bulbs were tested for length of life and the following results were obtained -

	Size	Sample mean	S.D
Type I	8	1234	36 hrs
Type II	7	1036	40 hrs

Table of value is 11.09

$n_1 = 8,\ n_2 = 7$

$\bar{x}_1 = 1234,\ \bar{x}_2 = 1036$

$S_1 = 36,\ S_2 = 40$

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{S} \times \sqrt{\frac{n_1 n_2}{n_1+n_2}}$$

$$= \frac{|1234-1036|}{40.7} \times \sqrt{\frac{56}{15}} = 9.39$$

$$S^2 = \frac{S_1^2 n_1 + S_2^2 n_2}{n_1+n_2-2} = \sqrt{1659}$$

$$\frac{1296 \times 8 + 1600 \times 7}{13} = \frac{10368 + 11200}{13} = 1659$$

$\boxed{S = 40.7}$

Hence, the solⁿ is accepted.

Q→ The following table refers to observation -

Sample I	25	30	28	34	24	20	13	32	22	38
Sample II	40	34	22	20	31	40	30	23	36	17

Analyse whether the sample have been drawn from the population of equal means.

F - test (Varian

Variance

$$\boxed{S_1^2 = \frac{n_1 S_1^2}{n_1 - 1}}$$

where $S_1 = \Sigma$

(1) Q→ Two random

are as follow

A 17 27

B 16 16

Test whether

population.

Q→ (2) A 28

B 29

Time serie

moving aver

suppose we

(1) 3 yearly

(2) 5 yearly

3/ F - test (Variance Ratio test) →

$$\text{Variance Ratio } (F) = \frac{S_1^2}{S_2^2} \quad (\text{if } S_1^2 > S_2^2)$$

$$= \frac{S_2^2}{S_1^2} \quad (\text{if } S_2^2 > S_1^2)$$

$$\boxed{S_1^2 = \frac{n_1 S_1^2}{n_1 - 1} \ \& \ S_2^2 = \frac{n_2 S_2^2}{n_2 - 1}}$$

where $S_1 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1}$ & $S_2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1}$

Q→ (1) Two random samples drawn from 2 normal populations are as follows -

A	17	27	18	25	27	29	13	17
B	16	16	20	27	26	25	21	

Test whether the sample are drawn from the same normal population.

Q→ (2)

A	28	30	32	33	31	29	34
B	29	30	30	24	27	28	

Time series → A set of data depending on time is called time series.

Moving average method →

suppose we have n term $y_1, y_2, y_3, y_4, \ldots y_n$ then

(1) 3 yearly moving average ⇒ $\frac{y_1 + y_2 + y_3}{3}$, $\frac{y_2 + y_3 + y_4}{3}$, $\frac{y_3 + y_4 + y_5}{3}$

(2) 5 yearly moving average ⇒ $\frac{y_1 + y_2 + y_3 + y_4 + y_5}{5}$, $\frac{y_2 + y_3 + y_4 + y_5 + y_6}{5}$

Q→ The following data are related to total sale. Show them graphically & given the trend value as 3 yearly moving average.

Year	Sales (Rs.)	Moving avg. total	Avg.
1993	3		
1994	4	15	5
1995	8	18	6
1996	6	21	7
1997	7	24	8
1998	11	27	9
1999	9	30	10
2000	10	33	11
2001	14	36	12
2002	12		

Now, we draw the graph