Unit → 4

Curve Fitting →

Principle (method) of least sequares → Suppose, we have the funct^n $y = ax^2 + bx + c$ by putting $x = x_1, x_2, \dots x_n$ we get the observed value $y_1, y_2, \dots y_n$ and $y_1', y_2', \dots y_n'$ are the expected values. By using method of least square we find the min values of constants a, b, c

$$\boxed{U = \sum (y_r - y_r')^2}$$

for min. values

$$\boxed{\frac{\partial U}{\partial a} = 0} \quad \boxed{\frac{\partial U}{\partial b} = 0} \quad \boxed{\frac{\partial U}{\partial c} = 0}$$

Type(1) Fitting the straight line →

Let the Eq^n of straight line is

$$\boxed{y = ax + b} \quad — ①$$

Calculate expected funct^n at (x_i, y_i) where $i = 1, 2, 3, \dots$

$$\boxed{y_i = ax_i + b} \quad — ②$$

By using method of least square

$$\boxed{U = \sum (y - y_i)^2} \;\&\; \frac{\partial U}{\partial a} = 0, \quad \frac{\partial U}{\partial b} = 0$$

$$U = \sum (y - ax_i - b)^2$$

$$\frac{\partial U}{\partial a} = 0$$

$$2\sum (y - ax_i - b)(-x_i) = 0$$

$$-2x_i \sum (y - ax_i - b) = 0$$

$$\sum (-y - ax_i - b) = 0$$

$$-\sum yx_i + \sum ax_i^2 + \sum bx_i = 0$$

$$\frac{\partial U}{\partial b} = 0$$

$$2\sum (y - ax_i - b)(-1) = 0$$

$$-2\sum (y - ax_i - b) = 0$$

$$-\sum y + a\sum x_i + \sum b = 0$$

$$\boxed{-\sum y + a\sum x_i + nb = 0}$$

Q→ To fit the straight line $y = a + bx$ for the following data-

x	0	5	10	15	20	25
y	12	15	17	22	24	30

$$y = a + bx \quad \text{— ①}$$

calculate at (x_i, y_i)

$$\boxed{y_i = a + bx_i}$$

Apply the method of least square

$$U = \sum (y - y_i)^2 \quad \& \quad \frac{\partial U}{\partial a} = 0, \ \frac{\partial U}{\partial b} = 0$$

$$U = \sum (y - a - bx_i)^2$$

$$\frac{\partial U}{\partial a} = 2\sum (y - a - bx_i)(-1) = 0$$

$$\Rightarrow -\sum y + \sum a + b\sum x_i = 0$$

$$-\sum y + na + b\sum x_i = 0$$

$$\boxed{\sum y - na - b\sum x_i = 0} \quad \text{— ①}$$

$$\frac{\partial U}{\partial b} = 0$$

$$2\sum (y - a - bx_i)(-x_i) = 0$$

$$(-x_i)(\sum y - \sum a - b\sum x_i) = 0$$

$$\sum(-x_i y + x_i a + bx_i^2) = 0$$

$$\boxed{-\sum x_i y + a\sum x_i + b\sum x_i^2 = 0} \quad \text{— ②}$$

x	y	xy	x^2
0	12	0	0
5	15	75	25
10	17	170	100
15	22	330	225
20	24	480	400
25	30	750	625
75	120	1805	1375

from Eq^n ①

$$\sum y - na - b\sum x_i = 0$$

$$120 - 6 \times a - b \times 75 = 0$$

$$120 - 6a - 75b = 0$$

$$6a + 75b = 120 \quad \text{— ③}$$

from Eq^n ②

$$-\sum x_i y + a\sum x_i + b\sum x_i^2 = 0$$

$-1805 + a \times 75 + b \times 1375 = 0$

$$75a + 1375b = 1805 \quad — (4)$$

$a = 11.2857$

$b = 0.6971$

$y = a + bx$

$= 11.2857 + 0.6971x$ Ans

Q→ By the method of least square find the straight line $y = a + bx$ that ~~bext~~ ~~ft~~ best fit the following data-

x	1	2	3	4	5
y	14	27	40	55	68

Given $y = a + bx$ — (1)

Put $x = x_i$ & $y = y_i$ in Eq^n (1)

$$\boxed{y_i = a + bx_i}$$

Apply the least square method:

$$U = \Sigma (y - y_i)^2$$

and $\frac{\partial U}{\partial a} = 0$, $\frac{\partial U}{\partial b} = 0$

$$U = \Sigma (y - a - bx_i)^2$$

$$\frac{\partial U}{\partial a} = 2\Sigma (y - a - bx_i)(-1) = 0$$

$$-\Sigma y + \Sigma a + b\Sigma x_i = 0$$

$$\boxed{-\Sigma y + na + b\Sigma x_i = 0} \quad — (2)$$

$$\frac{\partial U}{\partial b} = 2\Sigma (y - a - bx_i)(-x_i) = 0$$

$$\boxed{-\Sigma yx_i + a\Sigma x_i + \Sigma b x_i^2 = 0} \quad — (3)$$

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	55	220	16
5	68	340	25
15	204	748	55

from Eq^n (2)

$$-\Sigma y + na + b\Sigma x_i = 0$$

$$-204 + 5a + b \times 15 = 0$$

$$5a + 15b = 204 \quad — (4)$$

from Eq^n ③

$$-\sum y x_i + a\sum x_i + \sum b x_i^2 = 0$$

$$-748 + a \times 15 + b \times 55 = 0$$

$$15a + 55b = 748 \quad — ⑤$$

Hence the solⁿ is

$$a = 0 \quad , b = 13.6$$

$$y = a + bx$$

$$= 0 + 13.6x$$

$$= 13.6x$$

Q→

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

x	y	xy	x^2
0	1	0	0
1	1.8	1.8	1
2	3.3	6.6	4
3	4.5	13.5	9
4	6.3	25.2	16
10	16.9	47.1	30

from Eq^n ①

$$-\sum y + na + b\sum x_i = 0$$

$$-16.9 + 5a + b \times 10 = 0$$

$$5a + 10b = 16.9 \quad — ④$$

from Eq^n ②

$$-\sum y x_i + a\sum x_i + \sum b x_i^2 = 0$$

$$-47.1 + a \times 10 + b \times 30 = 0$$

$$10a + 30b = 47.1 \quad — ⑤$$

$$a = 0.72, \quad b = 1.33$$

Hence, the solⁿ is

$$y = a + bx$$

$$= 0.72 + 1.33x$$

Fitting an exponential curve of type $y = ae^{bx}$ →

Given → $y = ae^{bx}$

taking log on both sides

$\log_{10} y = \log_{10}(ae^{bx})$

$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$ ($\because \log mn = \log m + \log n$)

$\log_{10} y = \log_{10} a + bx \log_{10} e$

$\log_{10} y = \log_{10} a + (b \log_{10} e) x$

$$\boxed{Y = A + Bx}$$

Normal eq^n →

$\Sigma Y = nA + B\Sigma x$

$\Sigma xY = A\Sigma x + B\Sigma x^2$

$A = \log_{10} a$

Taking antilog

$$\boxed{a = (10)^A}$$

$B = a \log_{10} e$

$$\boxed{b = \frac{B}{\log_{10} e}}$$

Q → Find the curve of best fit of the type $y = ae^{bx}$ to the following data →

x	1	5	7	9	10
y	10	15	12	15	21

Given $y = ae^{bx}$

$\log_{10} y = \log_{10} a + (b \log_{10} e) x$

$$\boxed{Y = A + Bx}$$

$\Sigma Y = nA + B\Sigma x$ — ①

$\Sigma xY = A\Sigma x + B\Sigma x^2$ — ②

$Y = \log_{10} y$, $A = \log_{10} a$, $B = b \log_{10} e$

Now, we make the table → $n = 5$

x	y	$Y = \log_{10} y$	xY	x^2
1	10	1	1	1
5	15	1.1761	5.8805	25
7	12	1.0792	7.5544	49
9	15	1.1761	10.5849	81
10	21	1.3222	13.2220	100
32		5.7536	38.2418	256

$\Sigma Y = nA + B\Sigma X$

$5.7536 = 5A + B \times 32$

$5A + 32B = 5.7536$ — ③

$\Sigma XY = A\Sigma X + B\Sigma X^2$

$37.2418 = A \times 32 + B \times 256$

$32A + 256B = 38.2418$

$\boxed{A = 1.0984 \quad B = 0.0082}$ or $A = 0.9734$, $B = 0.0277$

Now, we find the solⁿ for the given curve -

$y = ae^{bx}$

$A = \log_{10} a$

$1.0984 = \log_{10} a$

Taking antilog

$a = (10)^{1.0984}$

$\boxed{a = 12.5430}$

$B = b\log_{10} e$

$0.0082 = b \times 0.4343$

$b = \dfrac{0.0082}{0.4343} \Rightarrow \boxed{b = 0.0189}$

Hence, the solⁿ is

$y = ae^{bx}$

$\boxed{y = 12.5e^{0.1x}}$ Ans

Q→ Determine the constant a & b by the method of least square such that $y = ae^{bx}$

x	2	4	6	8	10
y	4.077	11.048	30.128	81.897	222.62

Q→ Determine the curve for $P = mw + s$

P	12	15	21	25
w	50	70	100	120

Given $P = mw + s$

$\Sigma P = m\Sigma w + ns$ — ①

$\Sigma Pw = m\Sigma w^2 + s\Sigma w$ — ②

p	w	pw	w^2
12	50	600	2500
15	70	1050	4900
21	100	2100	10000
25	120	3000	14400
73	340	6750	31800

$$\Sigma p = m\Sigma w + n s$$

$$73 = m \times 34 + 4 \times s$$

$$\Rightarrow 34m + 4s = 73 \quad \text{— (3)}$$

$$\Sigma pw = m\Sigma w^2 + s\,\Sigma w$$

$$6750 = m \times 3180 + s \times 340$$

$$3180\,m + 340\,s = 6750 \quad \text{— (4)}$$

$$s = 2.2759$$

$$m = 0.1879$$

$$\boxed{p = 0.1879\,w + 2.2759}$$ Ans

Q→ Use the method of least square to fit the curve $y = \frac{c_0}{x} + c_1\sqrt{x}$ to the following table of values-

x	0.1	0.2	0.4	0.5	1.6	2.0
y	21	11	7	6	5	6

Given :→ $y = \frac{c_0}{x} + c_1\sqrt{x}$ — (1)

error at (x_i, y_i)

$$y_1 = \frac{c_0}{x_i} + c_1\sqrt{x_i} \quad \text{— (2)}$$

Apply the least square method

$$U = \Sigma(y - y_i)^2$$

$$U = \Sigma\left(y - \frac{c_0}{x_i} - c_1\sqrt{x_i}\right)^2$$

$$\frac{\partial U}{\partial c_0} = 2\Sigma\left(y - \frac{c_0}{x_i} - c_1\sqrt{x_i}\right)\left(\frac{-1}{x_i}\right) = 0$$

$$\Sigma\left(\frac{-y}{x_i} + \frac{c_0}{x_i^2} + \frac{c_1}{\sqrt{x_i}}\right) = 0$$

$$-\Sigma\frac{y}{x_i} + \frac{\Sigma c_0}{x_i^2} + c_1\Sigma\frac{1}{\sqrt{x_i}} = 0$$

$$\boxed{\frac{\Sigma y}{x_i} = c_0\Sigma\frac{1}{x_i^2} + c_1\Sigma\frac{1}{\sqrt{x_i}}} \quad \text{— (3)}$$

$$\frac{\partial U}{c_1} = 2\Sigma\left(y - \frac{c_0}{x_i} - c_1\sqrt{x_i}\right)\left(-\sqrt{x_i}\right) = 0$$

$$= \Sigma\left(-\sqrt{x_i}\,y + \frac{c_0}{\sqrt{x_i}} + c_1 x_i\right) = 0$$

$$\Rightarrow -\Sigma\sqrt{x_i}\,y + c_0\Sigma\frac{1}{\sqrt{x_i}} + c_1\Sigma x_i = 0$$

$$\boxed{\Sigma\sqrt{x_i}\,y = c_0\Sigma\frac{1}{\sqrt{x_i}} + c_1\Sigma x_i} \quad — ④$$

x	y	$\sqrt{x}$	$\sqrt{x}y$	y/x	x^2	$1/x^2$	$1/\sqrt{x}$
0.1	21	0.32	6.72	210	0.01	100	3.16
0.2	11	0.45	4.95	55	0.04	25	2.24
0.4	7	0.63	4.41	17.50	0.16	6.25	1.58
0.5	6	0.71	4.26	12	0.25	4	1.41
1.6	5	1.26	6.30	3.13	2.56	0.39	0.79
2.0	6	1.41	8.46	3	4	0.25	0.71
4.8			35.10	300.63		135.89	9.89

from Eq^n ③ →

$$300.63 = c_0 \times 135.89 + c_1 \times 9.89$$

$$135.89c_0 + 9.89c_1 = 300.63$$

from Eq^n ④ →

$$35.10 = c_0 \times 9.89 + c_1 \times 4.8$$

$$9.89c_0 + 4.8c_1 = 35.10$$

$$c_0 = 1.98$$

$$c_1 = 3.24$$

from Eq^n ① →

$$y = \frac{c_0}{x} + c_1\sqrt{x}$$

$$y = \frac{1.98}{x} + 3.24\sqrt{x} \quad Ans$$

Q → obtain a relation to the form $y = ab^x$ for the following data -

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	126.4

Given → $y = ab^x$ — ①

taking $\log_{10}$ on both sides

$$\log_{10} y = \log_{10} a + x\log_{10} b$$

$$\boxed{Y = A + Bx} \quad — ②$$

write the normal eq^n of eq^n ② using least square method.

$$\Sigma Y = nA + B\Sigma X \quad — ③$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2 \quad — ④$$

$$Y = \log_{10} y$$

x	y	$Y = \log_{10} y$	xy	x^2
2	8·3	0·92	1·84	4
3	15·4	1·19	3·57	9
4	33·1	1·52	6·08	16
5	65·2	1·81	9·05	25
6	126·4	2·10	12·60	36
20		7·54	33·1	90

from Eq^n ③

$$7·54 = 5A + B \times 20$$

$$5A + 20B = 7·5 \quad — ⑤$$

from Eq^n ④

$$33·1 = A \times 20 + B \times 90$$

$$20A + 90B = 33·1 \quad — ⑥$$

$$A = 0·3, \; B = 0·3$$

$A = \log_{10} a$

$0·3 = \log_{10} a$

Taking antilog

$a = 2·0$

$B = \log_{10} b \Rightarrow 0·3 = \log_{10} b$

Taking antilog

$b = 2·0$

from Eq^n ①

$$y = ab^x$$

$$y = 2·0 \times 2·0^x \quad \underline{Ans}$$

Regression Analysis

Regression analysis are used to find the relationship b/w dependent variable and independent variable.

Types of Regression

- Linear: $y = a + bx^{1}$
- Non linear: $y = a + bx^{2,4,\ldots}$
- multiple regression: $y = ax + bz^{2} + \cdots$

(1) Linear regression →

Derivation of Linear regression

(a) Line of regression of y on x → To find regression line of y on x we shall assume y as depend variable & x as independent variable.

Let $y = a + bx$ be the line as regression of y on x.

The error at i^{th} point is

$$y_i = a + bx_i$$

where (x_i, y_i) $(i = 0, 1, 2, \ldots)$ are observation.

According to the principle of least square, the constant a & b are selected in such a way the square of error is min.

$$U = \Sigma (y - y_i)^2$$

$$U = \Sigma (y - a - bx_i)^2 \quad \text{— (1)}$$

for min. values the condition is

$$\frac{\partial U}{\partial a} = 0 \quad \& \quad \frac{\partial U}{\partial b} = 0$$

The normal eq^n of eq^n (1) are

$$\Sigma y = na + b\Sigma x \quad \text{— (2)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{— (3)}$$

On solving eq^n (2) & (3) we get

$$a = \frac{\Sigma y}{n} - \frac{b\Sigma x}{n}$$

$$\boxed{a = \bar{y} - b\bar{x}} \quad \text{— (4)}$$

$$b = \frac{\Sigma xy}{\Sigma x} - \frac{a\Sigma x}{\Sigma x^2}$$

$$\boxed{b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}} \quad \text{— (5)}$$

Put $x = \bar{x}$ and $y = \bar{y}$ in $y = a + bx$

$$\boxed{\bar{y} = a + b\bar{x}} \quad \text{— (6)} \quad \text{where } a = y - bx$$

$$\bar{y} = y - bx + b\bar{x}$$

$$\boxed{y - \bar{y} = b(x - \bar{x})}$$

This is the eqn of regression of y on x where b is the regression coefficient.

Regression coefficient → The coefficient b is also written as

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

Regression line of x on y →

$$(x - \bar{x}) = b(y - \bar{y})$$

$$b = \frac{n\Sigma yx - \Sigma y \Sigma x}{n\Sigma y^2 - (\Sigma y)^2}$$

$$\boxed{b_{xy} = \frac{n\Sigma yx - \Sigma y \Sigma x}{n\Sigma y^2 - (\Sigma y)^2}}$$

Q → Find the two line of regression & coefficient of regression for the data given below-

$n = 18$, $\Sigma x = 12$, $\Sigma y = 18$, $\Sigma x^2 = 60$, $\Sigma y^2 = 96$, $\Sigma xy = 48$.

Line of regression of y on x

$$(y - \bar{y}) = b(x - \bar{x}) \quad \text{— (1)}$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{18}{18} = 1$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{12}{18} = \frac{2}{3}$$

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{n\Sigma x\Sigma y - \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2} = \frac{18 \times 12 \times 18 - 48}{18 \times 60 - 144}$$

$$= \frac{18 \times 48 - 12 \times 18}{18 \times 60 - 144} = \frac{3888 - 48}{1080 - 144} = \frac{3840}{936} = 4.103 \text{ Ans}$$

$= y - bx$

ere b is the

written as

regression for

18.

8

3 Ans

$$b = \frac{648}{936} = 0.692$$

Put the value into Eq^n ①

$$y - 1 = 4.103\left(x - \frac{2}{3}\right) \qquad y - 1 = 0.692\left(x - \frac{2}{3}\right)$$

$$y - 1 = \frac{4.103}{3}(3x - 2)$$

$$y - 1 = 1.3677(3x - 2)$$

$$\boxed{y - 1 = 1.368(3x - 2)} \text{ Ans}$$

line of regression of x on y

$$(x - \bar{x}) = b(y - \bar{y})$$

$$b = \frac{n\Sigma x \Sigma y - \Sigma xy}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{18 \times 12 \times 18 - 48}{18 \times 96 - (18)^2}$$

$$= \frac{3840}{1728 - 324}$$

$$b = \frac{3840}{1404} = 2.735$$

Put the value into Eq^n ②

$$x - \frac{2}{3} = 2.735(y - 1)$$

$$x - 0.667 = 2.735(y - 1) \text{ Ans}$$

formulaes

(1) $\Sigma(x - \bar{x}) = 0$

(2) std. derivation (S.D.) →

$$\sigma x = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

$$\sigma y = \sqrt{\frac{\Sigma(y - \bar{y})^2}{n}}$$

(3) varience $= (S.D.)^2$

(4) coefficient of corelation →

$$r = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{n \sigma x \sigma y}$$

(5) $b_{xy}\, b_{yx} = r^2$

Q→ If the regression coefficients are 0.8 & 0.2 what would be the value of coefficient of correlation.

Given → $b_{xy} = 0.8$
$b_{yx} = 0.2$

We know that,
$r^2 = b_{xy}\, b_{yx}$
$= 0.8 \times 0.2$
$r^2 = 0.16$

$\boxed{r = \pm 0.4}$ [Neglect the -ve value because r is never -ve]

Q→ Calculate linear regression coefficient from the following data-

x	y	xy	x^2	y^2
1	3	3	1	9
2	7	14	4	49
3	10	30	9	100
4	12	48	16	144
5	14	70	25	196
6	17	102	36	289
7	20	140	49	400
8	24	192	64	576
36	107	599	204	1763

Regression coefficient of y on x

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{8 \times 599 - 36 \times 107}{8 \times 204 - (36)^2}$$

$$= 2.7976 \text{ Ans}$$

Regression coefficient of x on y

$$b_{xy} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

$$= \frac{8 \times 599 - 36 \times 107}{8 \times 1763 - (107)^2}$$

$$= 0.3540 \text{ Ans}$$

8→ The following table gives age(x) in years of cars & annual maintenance cost(y) in hundred rupees-

x	1	3	5	7	9
y	15	18	21	23	22

Estimate the maintenance cost for a 4 year old car after finding the regression Eq^n.

9→ obtain the line of regression of y on x for the data given below-

x	1.53	1.78	2.60	2.95	3.42
y	33.50	36.30	40.00	45.80	53.50

9→ In a partially destroyed laboratory record of an analysis of a correlation data, the following results are given -

Varience of $x = 9$

Regression line → $8x - 10y + 66 = 0$

$40x - 18y = 214$

what were (a) the mean value of x & y

(b) the S.D. of y and coefficient of correlation b/w x & y.

x on y → $8x - 10y + 66 = 0$

$8x = -66 + 10y$

$\boxed{x = -8.25 + 1.25y}$ $\qquad x = a + by$

$\boxed{b_{xy} = 1.25}$

y on x → $18y = -214 + 40x$

$y = -11.88 + 2.22x$

$\boxed{b_{yx} = 2.22}$

Let, the Eq^n x on y & y on x are passes from $(\bar{x}, \bar{y})$

$\bar{x} = -8.25 + 1.25\bar{y} \Rightarrow \bar{x} - 1.25\bar{y} = -8.25$

$\bar{y} = -11.88 + 2.22\bar{x} \Rightarrow 2.22\bar{x} - \bar{y} = 11.88$

$\boxed{\bar{x} = 13.01}$

$\boxed{\bar{y} = 17.01}$

$$r^2 = b_{yx} \cdot b_{xy}$$

$$= 2.22 \times 1.25$$

$$\boxed{r = 1.67} \text{ Ans}$$

$$\text{Variance} = (S.D.)^2$$

$$9 = (S.D.)^2$$

$$\boxed{\begin{array}{c} S.D = 3 \\ \hline \sigma x = 3 \end{array}} \text{ Ans}$$

Non-linear regression (Polynomial in 2 degree / parabola curve) :-

$$\boxed{y = a + bx + cx^2}$$

Q→ Fit a second degree parabola to the following data -

x	0	1	2
y	1	6	17

we know that second degree parabola eqⁿ is

$$\boxed{y = a + bx + cx^2} \quad \text{—} ①$$

error at i^{th} point

$$\boxed{y_i = a + bx_i + cx_i^2} \quad \text{—} ②$$

Find the normal eqⁿs using method of least square -

$$U = \Sigma (y - y_i)^2$$

$$\boxed{U = \Sigma (y - a - bx_i - cx_i^2)^2}$$

for min. values of a, b & c

$$\boxed{\frac{\partial U}{\partial a} = \frac{\partial U}{\partial b} = \frac{\partial U}{\partial c} = 0}$$

The normal eqⁿ are -

$$\left.\begin{array}{l} \Sigma y = na + b\Sigma x + c\Sigma x^2 \\ \Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \\ \Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \end{array}\right]$$

x	y	x^2	xy	x^3	x^2y	x^4
0	1	0	0	0	0	0
1	6	1	6	1	6	1
2	17	4	34	8	68	16
3	24	5	40	9	74	17

Put these values in normal eq^{n}s

$$24 = 3a + 3b + 5c$$
$$40 = 3a + 5b + 9c$$
$$74 = 5a + 9b + 17c$$
$$a = 1,\ b = 2,\ c = 3 \text{ Ans}$$

Q→ fit a parabola $y = ax^2 + bx + c$

x	0	1	2	3	4
y	1	4	10	17	30

Given→ $y = ax^2 + bx + c$ — ①

find the normal eq^n using method of least square -

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc \text{ — ②}$$
$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x \text{ — ③}$$
$$\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 \text{ — ④}$$

x	y	x^2	xy	x^3	x^2y	x^4
0	1	0	0	0	0	0
1	4	1	4	1	4	1
2	10	4	20	8	40	16
3	17	9	51	27	153	81
4	30	16	120	64	480	256
10	62	30	195	100	~~433~~ 677	354

$$62 = 30a + 10b + 5c \text{ — ⑤}$$
$$195 = 100a + 30b + 10c \text{ — ⑥}$$
$$677 = 354a + 100b + 30c \text{ — ⑦}$$

$$a = 1.50,\ b = 1.10,\ c = 1.20 \text{ Ans}$$

multiple regression→

$$y = a + bx + cz$$

Q→ obtain a regression plane by using multiple linear regression to fit the data given below-

x	1	2	3	4
y	0	1	2	3
z	12	18	24	30

Given→ $y = a + bx + cz$ — ①

error at i^{th} point-

$$y_i = a + bx_i + cz_i$$

apply, the least square method

$$U = \Sigma (y - y_i)^2$$

$$= \Sigma (y - a - bx_i - cz_i)^2$$

for min. values of a, b, c

$$\frac{\delta U}{\delta a} = 0, \quad \frac{\delta U}{\delta b} = 0, \quad \frac{\delta U}{\delta c} = 0$$

$$\frac{\delta U}{\delta a} = 0$$

$$2\Sigma(y - a - bx_i - cz_i)(-1) = 0$$

$$\Sigma(-y + a + bx_i + cz_i) = 0$$

$$-\Sigma y + na + b\Sigma x + c\Sigma z = 0$$

similarly, we can find other two eq^n's

$$\frac{\delta U}{\delta b} = 0$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma xz \text{ — ③}$$

$$\frac{\delta U}{\delta c} = 0$$

$$\Sigma yz = a\Sigma z + b\Sigma xz + c\Sigma z^2 \text{ — ④}$$

x	y	z	xy	x^2	xz	yz	z^2
1	0	12	0	1	12	0	144
2	1	18	2	4	36	18	324
3	2	24	6	9	72	48	576
4	3	30	12	16	120	90	900
10	6	84	20	30	240	156	1944

$$6 = 4a + 10b + 84c \quad \text{— (5)}$$

$$20 = 10a + 30b + 240c \quad \text{— (6)}$$

$$156 = 84a + 240b + 1944c \quad \text{— (7)}$$

$$a = 1.5, \; b = 3.5, \; c = -0.4 \quad \text{Ans}$$