Unit 2

Given $y = f(x)$

x	x_0	x_1	x_2	------	x_n
$f(x)$	$f(x_0), y_0$	y_1	y_2	------	y_n

The process of finding the value of y b/w the range (i.e., x_0, x_n) is called interpolation and finding the value of y outside the range is called extrapolation.

Finite differences and difference tables:

(1) **Forward difference operator:** It is denoted by Δ (delta)

$$\boxed{\Delta y_n = y_{n+1} - y_n} \quad \text{where } n = 0, 1, 2, 3, \ldots$$

$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\Delta y_2 = y_3 - y_2$$
$$\vdots$$

forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		$\Delta y_0 = y_1 - y_0$			
x_1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$		
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
x_3	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		
		$\Delta y_3 = y_4 - y_3$			
x_4	y_4				

Q→ construct the forward difference table, given that -

x	5	10	15	20	25	30
y	9962	9848	9659	9397	9063	8660

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
5	9962					
		-114				
10	9848		-75			
		-189		2		
15	9659		-73		-1	
		-262		1		3
20	9397		-72		2	
		-334		3		
25	9063		-69			
		-403				
30	8660					

(2) Backward difference operator → It is denoted by ~~nebla~~ ∇ (nebla)

$$\boxed{\nabla y_n = y_n - y_{n-1}} \quad \text{where } n = 1, 2, 3, \ldots$$

$\nabla y_1 = y_1 - y_0$

$\nabla y_2 = y_2 - y_1$

⋮

Backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		$\nabla y_1 = y_1 - y_0$			
x_1	y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$		
		$\nabla y_2 = y_2 - y_1$		$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$	
x_2	y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$		$\nabla^4 y_4 = \nabla^3 y_4 - \nabla^3 y_3$
		$\nabla y_3 = y_3 - y_2$		$\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$	
x_3	y_3		$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$		
		$\nabla y_4 = y_4 - y_3$			
x_4	y_4				

Q→ Construct the backward difference table, given that

$x_{20} = 400, \ x_{30} = 600, \ x_{40} = 800, \ x_{50} = 1100$

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
20	400			
		200		
30	600		0	
		200		100
40	800		100	
		300		
50	1100			

Missing term technique →

Result → If n values are known that a f^n can be represented by $n-1^{th}$ degree polynomial.

Q→ Find the missing value of the following data -

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	7				
		$y - 7$			
2	$f(y)$		$20 - 2y$		
		$13 - y$		$3y - 25$	
3	13		$-5 + y$		$-4y + 38$
		8		$13 - y$	
4	21		8		
		6			
5	37				

since, there are 4 values are known. Hence, the f f(x) be represented by 3rd degree polynomial

$$\Rightarrow \Delta^4 f(x) = 0$$

$$-4y + 38 = 0$$

$$-4y = -38$$

$$y = \frac{38}{4} = 9.5$$

Q→ Given→ $\log 100 = 2$

$\log 101 = 2.0043$

$\log 103 = 2.01128$

$\log 104 = 2.0170$

find $\log 102$?

Let $\log 102 = y$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
log100	2				
		0.0043			
log101	2.0043		$y - 2.0086$		
		$y - 2.0043$		$-3y + 6.0257$	
log102	y		$4.0171 - 2y$		$6y - 12.0514$
		$2.0128 - y$		$3y + 6.0257$	
log103	2.0128		$y - 2.0086$		
		0.0042			
log104	2.0170				

Hence, there are 4 values are known. Hence y can be represented by 3rd degree polynomial

$$\Delta^4 y = 0$$

$$6y - 12.0514 = 0$$

$$6y = 12.0514$$

$$y = \frac{12.0514}{6} = 2.0086$$

Q→ Find the missing values-

x	45	50	55	60	65
y	3	?	2	?	-2.4

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
45	3				
		$y_1 - 3$			
50	y_1		$5 - 2y_1$		
		$2 - y_1$		$3y_1 + y_2 - 9$	
55	2		$y_1 + y_2 - 4$		$-4y - 4y_2 + 12.6$
		$y_2 - 2$		$-y_1 - 3y_2 + 3.6$	
60	y_2		$-2y_2 - 0.4$		
		$-2.4 - y_2$			
65	-2.4				

since, there are 3 values are known. Hence f^n can be represen- -ted by 2^{nd} degree polynomial.

$$\Rightarrow \boxed{\Delta^3 y = 0}$$

put E_q^n

$$3y_1 + y_2 = 9$$
$$-y_1 - 3y_2 = -3.6$$

~~Mode → E_q^n → 2~~
~~unknown 2~~

$a_1 = 3$, $a_2 = 1$, $a_3 = -9$
$b_1 = -1$, $b_2 = -3$, $b_3 = 3.6$
$y_1 = -2.9250$
$y_2 = +0.2250$

Interpolation with Equal interval →

x	40	42	50	57
y	100	200	150	600

Interpolation →

Interpolation with equal interval
- Newton's forward diff. formula
- Newton's backward diff. formula
- Central diff. formula
  - Gauss forward
  - Gauss backward
  - Sterling
  - Besslers
  - Laplace Evertte's

Interpolation with unequal interval
- ~~Language~~ Lagrange formulla
- Newton's devided
- Hermitte's formulla

Newton's forward difference formulla →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		Δy_0	$\Delta^2 y_0$		
x_1	y_1			$\Delta^3 y_0$	
		Δy_1			$\Delta^4 y_0$
x_2	y_2		$\Delta^2 y_1$		
				$\Delta^3 y_1$	
		Δy_2			
x_3	y_3		$\Delta^2 y_2$		
		Δy_3			
x_4	y_4				

$$y(x) = y_0 + \frac{u\Delta y_0}{1!} + \frac{u(u-1)\Delta^2 y_0}{2!} + \frac{u(u-1)(u-2)\Delta^3 y_0}{3!} + \frac{u(u-1)(u-2)(u-}{4!}$$

where $u = \frac{x_r - x_0}{h}$

Q→ Find the value of sin 52° from the given table -

Q	45	50	55	60
sinQ	0.7071	0.7660	0.8192	0.8660

Let $Q = x$ & $y = \sin Q$ & we multiply the value of y by 10^4

x	$y(\sin Q \times 10^4)$	Δy	$\Delta^2 y$	$\Delta^3 y$
45	7071			
		589		
50	7660		-57	
		532		-7
55	8192		-64	
		468		
60	8660			

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)\Delta^3 y_0}{3!} + \cdots$$

(by using Newton forward diff. formulla)

where, $u = \frac{x - x_0}{h}$

$= \frac{52-45}{5} = \frac{7}{5} = 1.4$

$$y(52) = 7071 + 1.4 \times 589 + \frac{1.4(1.4-1)(-57)}{2} + \frac{(1.4)(1.4-1)(1.4-2)(-7)}{6}$$

$= 7880.032/10000$

$= .7880$

Q→

x	0	1	2	3
$f(x)$	1	10	1	2

x	$f(x)$	Δx	$\Delta^2 x$	$\Delta^3 x$
0	1			
		9		
1	10		-18	
		-9		28
2	1		10	
		1		
3	2			

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)\Delta^3 y_0}{3!}$$

where, $u = \frac{x - x_0}{h} = \frac{x-0}{1} = x$

$\boxed{u = x}$

$$= 1+9x+\frac{x(x-1)}{2}(-18)+\frac{x(x-1)(x-2)}{6}\times 28$$

$$= 1+9x-9x(x-1)+\frac{14}{3}x(x-1)(x-2)$$

$$= \frac{3+27x-27x(x-1)+14x(x-1)(x-2)}{3}$$

$$= \frac{3+27x-27x^2+27x+(14x^2-14x)x-2}{3}$$

$$= \frac{3+27x-27x^2+27x+14x^3-14x^2-28x^2+28x}{3}$$

$$= \frac{14x^3-69x^2+55x+31}{3}$$

Newton's backward difference formulla →

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		∇y_1			
x_1	y_1		$\nabla^2 y_2$		
				$\nabla^3 y_3$	
x_2	y_2	∇y_2	$\nabla^2 y_3$		$\nabla^4 y_4$
x_3	y_3	∇y_3	$\nabla^2 y_4$	$\nabla^3 y_4$	
x_4	y_4	∇y_4			

$$y(x) = y_n+u\nabla y_n+\frac{u(u+1)}{2!}\nabla^2 y_n+\frac{u(u+1)(u+2)}{3!}\nabla^3 y_n+\cdots$$

where $u=\frac{x-x_n}{h}$

Q→ The population of a town was as given estimate the population for the year 1925.

year(x)	1891	1901	1911	1921	1931
population(y)	46	66	81	93	101

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
		20	-5		
1901	66			2	
		15	-3		-3
1911	81			-1	
		12	-4		
1921	93				
		8			
1931	101				

using Newton's backward formulla

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

where, $u = \frac{x - x_n}{h}$

$$= \frac{1925-1931}{10} = \frac{-6}{10} = -0.6$$

$$= 101 + (-0.6)8 + \frac{(-0.6)(-0.6+1)(-4)}{2} + \frac{(-0.6)(-0.6+1)(-0.6+2)(-}{6}$$

$$+ \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-3)}{24}$$

$$= 101 + (-4.8) + \frac{0.96}{2} + \frac{0.336}{6} + \frac{2.4192}{24}$$

$$= \frac{2424 - 115.2 + 11.52 + 1.344 + 2.4192}{24}$$

$$= \frac{2324.0832}{24} = 96.8368$$

Q→ find the missing value of following data→

x	1	2	3	4	5
f(x)	7	y	13	21	37

x	f(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	7				
		y-7			
2	4		20-2y		
		13-y		3y-25	
3	13		y-5		-4y+38
		8		13-y	
4	21		8		
5	37	16			

$$-4y + 38 = 0$$

$$-4y = -38$$

$$y = \frac{38}{4} = 9.5$$

Q→ consider the following table

x	3	4	5	6	7
f(x)	3	6.6	15	22	35

obtain interpolating polynomial or as degree 2 or less usin

Newton's backward difference interpolating method.

x	f(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
3	3				
4	6.6	3.6			
5	15	8.4	4.8		
6	22	7	-1.4	-6.2	
7	35	13	6	7.4	13.6

where, $u = \dfrac{x - x_n}{h} = \dfrac{+x-7}{1} = x-7$

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_n$$

$$= 35 + (x-7)\times 13 + \frac{(x-7)(x-7+1)\times 6}{2} + \frac{(x-7)(x-7+1)(x-7+2)(7.4)}{6} + \frac{(x-7)(x-7+1)(x-7+2)(x-7+3)(13.6)}{24}$$

$$= 35 + 13x - 91 + \frac{(x^2 - 7x + x - 7x + 49 - 7)6}{2} + \frac{(x^2 - 7x + x - 7x + 49 - 7)(x-7+2)(7.4)}{6} + \frac{(x^2 - 7x + x - 7x + 49 - 7)(x-7+2)(x-7+3)(13.6)}{24}$$

$$= 35 + 13x - 91 + \frac{(6x^2 - 42x + 6x - 42x + 294 - 42)}{2} + (x^3 - 7x^2 + 2x^2$$

$$= 35 + (x-7)\times 13 + \frac{(x-7)(x-6)\times 6}{2} + \frac{(x-7)(x-6)(x-5)\,7.4}{6} + \frac{(x-7)(x-6)(x-5)(x-4)(13.6)}{24}$$

$$= 35 + (x-7)13 + \frac{(x^2 - 6x - 7x + 42)6}{2} + \frac{(x^2 - 6x - 7x + 42)(x-5)\,7.4}{6} + \frac{(x^2 - 6x - 7x + 42)(x-5)(x-4)(13.6)}{24}$$

$$= 35 + 13x - 91 + \frac{(x^2 - 13x + 42)6}{2} + \frac{(x^2 - 13x + 42)(x-5)(7.4)}{6} + \frac{(x^2 - 13x + 42)(x-5)(x-4)(13.6)}{24}$$

$$= 35+13x-91+\frac{6x^2-78x+252}{2}+\frac{(x^3-13x^2+42x-5x^2+65x-\dots}{6}$$

$$+\frac{(x^3-13x^2+42x-5x^2+65x-210)(x-4)(13\cdot 6)}{24}$$

$$= 35+13x-91+\frac{6x^2-78x+252}{2}+\frac{(x^3-18x^2+107x-210)(7\cdot 4)}{6}$$

$$+\frac{(x^3-18x^2+107x-210)(x-4)(13\cdot 6)}{24}$$

$$= 35+13x-91+\frac{6x^2-78x+252}{2}+\frac{7\cdot 4x^3-133\cdot 2x^2+791\cdot 8x-1554}{6}$$

$$+\frac{(x^4-18x^3+107x^2-210x-4x^3+72x^2-428x+840)(13\cdot 6)}{24}$$

$$= 35+13x-91+\frac{6x^2-78x+252}{2}+\frac{7\cdot 4x^3-133\cdot 2x^2+791\cdot 8x-\dots}{6}$$

$$+\frac{(x^4-22x^3+179x^2-638x+840)(13\cdot 6)}{24}$$

$$= 35+13x-91+\frac{6x^2-78x+252}{2}+\frac{7\cdot 4x^3-133\cdot 2x^2+791\cdot 8x-155\dots}{6}$$

$$+\frac{13\cdot 6x^4-299\cdot 2x^3+2434\cdot 4x^2-8676\cdot 8x+11424}{24}$$

$$= \frac{840+312x-2184+72x^2-936x+3024+29\cdot 6x^3-532\cdot 8x^2+3167\cdot 2x-6\dots}{24}$$

$$+13\cdot 6x^4-299\cdot 2x^3+2434\cdot 4x^2-8676\cdot 8x+11424$$

$$= \frac{13\cdot 6x^4-269\cdot 6x^3+1973\cdot 6x^2-6133\cdot 6x+6888}{24}$$

Difference of a polynomial →

Theorem → The n^{th} difference of a polynomial of n^{th} degree ar… constant and all higher order difference are …

or

Prove that → $\Delta^n f(x) = \text{constant}$

also $\Delta^{n+1} f(x) = 0$

let we consider a polynomial of n^{th} degree

$$\boxed{f(x) = ax^n + bx^{n-1} + cx^{n-2} + \dots + kx + l} \quad \text{—— (1)}$$

where a, b, c, k, l are constant. we know that,

$$\Delta f(x) = f(x+h) - f(x)$$

put in E_q^n ①

$$f(x+h) = [a(x+h)^n + b(x+h)^{n-1} + c(x+h)^{n-2} + \cdots k(x+h) + l] - [ax^n + bx^{n-1} + cx^{n-2} + \cdots + kx + l]$$

$$= [a(x+h)^n - ax^n] + [b(x+h)^{n-1} - bx^{n-1}] + [c(x+h)^{n-2} - cx^{n-2}] + \cdots + [k(x+h) - kx] + [l - l]$$

$$= a[(x+h)^n - x^n] + b[(x+h)^{n-1} - x^{n-1}] + c[(x+h)^{n-2} - x^{n-2}] + \cdots + kh$$

Since, $a[(x+h)^n - x^n]$

$\Rightarrow a[x^n(1+\frac{h}{x})^n - x^n]$ expand by binomial theorem

$$\Rightarrow ax^n[(1+\frac{h}{x})^n - 1]$$

$$\Rightarrow ax^n[\cancel{1} + \frac{nh}{x} + \frac{n(n-1)}{2}\frac{h^2}{x^2} + \cdots - \cancel{1}]$$

$$= ax^n\frac{nh}{x} + \frac{ax^n n(n-1)}{2}\frac{h^2}{x^2} + \cdots$$

$$= anhx^{n-1} + \frac{ax^{n-2}n(n-1)}{2}h^2 + \cdots$$

$$\boxed{\Delta f(x) = anhx^{n-1} + b'x^{n-2} + c'x^{n-3} + \cdots + k'x + l'} \quad \text{—— ②}$$

where $(b', c', \cdots k', l')$ are new constant coefficient.

Therefore, the first difference represent a polynomial of $n-1$ degree.

$$\Delta^2 f(x) = \Delta f(x+h) - \Delta f(x)$$

$$= [anh(x+h)^{n-1} + b'(x+h)^{n-2} + c'(x+h)^{n-3} + \cdots + k'(x+h) + l'] - [anhx^{n-1} + b'x^{n-2} + c'x^{n-3} + \cdots + k'x + l']$$

$$= anh[(x+h)^{n-1} - x^{n-1}] + b'[(x+h)^{n-2} - x^{n-2}] + \cdots + k'h + 0$$

$$= anhx^{n-1}[(1+\frac{h}{x})^{n-1} - 1] + b'x^{n-2}[(1+\frac{h}{x})^{n-2} - 1] + \cdots k'h$$

$= anhx^{n-1}[(1+\frac{h}{x})^{n-1} - 1]$ expand by binomial theorem

$$= anhx^{n-1}(\cancel{1} + (n-1)\frac{h}{x} + \frac{(n-1)(n-1-1)}{2}\frac{h^2}{x^2} + \cdots - \cancel{1}]$$

$$= anhx^{n-1}(n-1)\frac{h}{x} + \cdots$$

$= an(n-1)h^2x^{n-2} + ---$

$\Rightarrow \boxed{\Delta^2 f(x) = an(n-1)h^2x^{n-2} + b''x^{n-3} + c''x^{n-4} + --- + k''x + l''}$

where (b'', c'', --- k'', l'') are constant. Therefore, the second difference of polynomial represent the n-2 degree poly.. similarly, the n^{th} differenca of polynomial represent polynomial of (n-n) degree i.e., constant term.

Therefore, $\Delta^n f(x) = \text{Constant}$

$\left.\begin{array}{l}\Delta^n f(x) = \text{Constant} \\ \Delta^{n+1} f(x) = 0\end{array}\right\}$ Hence Proved

Q→ The following table gives the marks secured by 100 studen.. the CBNST subject.

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	25	35	22	11	7

use Newton's forward formulla to find the no. of studen.. who secured less than 55 marks.

x	less than x	No. of stu.	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
30-40	40	25				
			35			
40-50	50	60		-13		
					2	
50-60	60	82	22			
				-11		5
60-70	70	93	11		7	
				-4		
70-80	80	100	7			

$x = 55$

$u = \frac{x - x_0}{h} = \frac{55-40}{10} = \frac{15}{10} = 1.5$

$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)\Delta^2 y_0}{2!} + \frac{u(u-1)(u-2)\Delta^3 y_0}{3!} + \frac{u(u-1)(u-2)(u-3)\Delta^4 y_0}{4!}$

$= 25 + 1.5 \times 35 + \frac{1.5(1.5-1) \times -13}{2} + \frac{1.5(1.5-1)(1.5-2)2}{6} + \frac{1.5(1.5-1)(1.5-2)...}{24}$

$= 25 + 52.5 + \frac{(-9.75)}{2} + \frac{(-0.75)}{6} + \frac{2.8125}{24}$

$= 25 + 52.5 - \frac{9.75}{2} - \frac{0.75}{6} + \frac{2.8125}{24}$

$= \frac{600 + 1260 - 117 - 3 + 2.8125}{24}$

$$= \frac{1742.8125}{24}$$

$$= 72.6$$

Interpolation with unequal interval →

Lagrange's interpolation formulla →

x	y
x_0	y_0
x_1	y_1
x_2	y_2
x_3	y_3

$y = ?$ at $x = ?$

$y(x) = ?$

$$y_{(x)} = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + - - -$$

Q → Use Lagrange's interpolation formulla find $y(10)$ from following table.

x	y
5	12
6	13
9	14
11	16

Apply Lagrange's interpolation formulla

$$y_{(x)} = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y_{(10)} = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} 13 +$$

$$\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} 16$$

$$= \frac{(4)(1)(-1)}{(-1)(-4)(-6)} 12 + \frac{(5)(1)(-1)}{(1)(-3)(-5)} 13 + \frac{(5)(4)(-1)}{(4)(3)(-2)} 14 + \frac{(5)(4)}{(6)(5)}$$

$$= \frac{(-4)\times 12}{-24} + \frac{(-5)\times 13}{15} + \frac{(-20)\times 14}{-24} + \frac{20 \times 16}{60}$$

$$= \frac{\cancel{-}48}{\cancel{-}24} - \frac{65}{15} + \frac{280}{24} + \frac{320}{60}$$

$$= \frac{240 - 520 + 1400 + 640}{120} = \frac{1760}{120} = 14.6$$

Q① → Find the unique polynomial $p(x)$ of degree 2 such that $p(1)=1$, $p(3)=27$, $p(4)=64$

x	$f(x)$
1	1
3	27
4	64

$$y(x) = \frac{(x-x_1)(x-x_2)\,y_0}{(x_0-x_1)(x_0-x_2)} + \frac{(x-x_0)(x-x_2)\,y_1}{(x_1-x_0)(x_1-x_2)} + \frac{(x-x_0)(x-x_1)\,y_2}{(x_2-x_0)(x_2-x_1)}$$

$$y(x) = \frac{(x-3)(x-4)1}{(1-3)(1-4)} + \frac{(x-1)(x-4)27}{(3-1)(3-4)} + \frac{(x-1)(x-3)}{(4-1)(4-3)}64$$

$$= \frac{(x^2-7x+12)}{-2\times-3} + \frac{(x^2-5x+4)27}{2\times-1} + \frac{(x^2-4x+3)}{3\times1}64$$

$$= \frac{(x^2-7x+12)}{6} - \frac{27}{2}(x^2-5x+4) + \frac{64}{3}(x^2-4x+3)$$

$$= \frac{x^2-7x+12-81(x^2-5x+4)+128(x^2-4x+3)}{6}$$

$$= \frac{x^2-7x+12-81x^2+405x-324+128x^2-512x+384}{6}$$

$$= \cancel{129x^2}\ \frac{48x^2-114x+72}{6}$$

$$= 8x^2-19x+12$$

Q① → The funct^n $y=f(x)$ is given at the points (7, 3), (8, 1), (9, 1), (10, 9). Find the values of y for $x=9.5$ using Lagrange's intepolation formulla.

Q② → Find the Lagrange interpolating polynomial for the data given below →

$x_0=1$, $x_1=2.5$, $x_2=4$, $x_3=5.5$

$f(x_0)=4$, $f(x_1)=7.5$, $f(x_2)=13$, $f(x_3)=17.5$

x	$f(x)$
1	4
2.5	7.5
4	13
5.5	17.5

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-3.5)(x-4)(x-5.5)}{(2-3.5)(2-4)(2-5.5)} 4 + \frac{(x-2)(x-\cancel{3.5}\,4)(x-5.5)}{(3.5-2)(3.5-4)(3.5-5.5)} 7.5$$

$$+ \frac{(x-2)(x-3.5)(x-5.5)}{(4-2)(4-3.5)(4-5.5)} 13 + \frac{(x-2)(x-3.5)(x-4)}{(5.5-2)(5.5-3.5)(5.5-4)} 17.5$$

$$= \frac{(x^2-4x-3.5x+14)(x-5.5)4}{-7} + \frac{(x^2-4x-2x+8)(x-5.5)7.5}{-1}$$

$$+ \frac{(x^2-3.5x-2x+7)(x-5.5)13}{1} + \frac{(x^2-3.5x-2x+7)(x-4)17.5}{7}$$

$$= \frac{(x^2-7.5x+14)(x-5.5)4}{-7} + \frac{(x^2-6x+8)(x-5.5)7.5}{-1}$$

$$+ \frac{(x^2-5.5x+7)(x-5.5)13}{1} + \frac{(x^2-5.5x+7)(x-4)17.5}{7}$$

$$= \frac{(x^3-5.5x^2-7.5x^2+41.25x+14x-77)4}{-7} + \frac{(x^3-5.5x^2-6x^2+33x+8\ldots}{-1}$$

$$+ \frac{(x^3-5.5x^2-5.5x^2+30.25x+7x-38.5)13}{1} + \frac{(x^3-4x^2-5.5x^2+22x+7x-2\ldots}{7}$$

$$\cancel{= (x^3-13x^2+37.25x-38.5)13}$$

$$= \frac{(x^3-13x^2+55.25x-77)4}{-7} + \frac{(x^3-11.5x^2+41x-44)7.5}{-1}$$

$$+ \frac{(x^3-11x^2+37.25x-38.5)13}{1} + \frac{(x^3-9.5x^2+29x-28)17.5}{7}$$

$$= \frac{4x^3-52x^2-221x-308}{-7} + \frac{7.5x^3-86.25x^2+307.5x-330}{-1}$$

$$+ \frac{13x^3-143x^2+484.25x-500.5}{1} + \frac{17.5x^3-166.25x^2+507.5x-\ldots}{7}$$

$$= \frac{-4x^3+52x^2+221x+308}{7} - \frac{7.5x^3+86.25x^2-307.5x+3\ldots}{1}$$

$$+ \frac{13x^3-143x^2+484.25x-500.5}{1} + \frac{17.5x^3-166.25x^2+507.5x-49\ldots}{7}$$

$$= \frac{-4x^3+52x^2+221x+308-52.5x^3+603.75x^2-2152.5x+2310}{7}$$

$$\underline{+91x^3-1001x^2+3389.75x-3503.5+17.5x^3-166.25x^2+507.5x-490}$$

$$= \frac{52x^3-511.5x^2+1965.75x-1375.5}{7}$$

Sol^n (1) →

x	$f(x)$
7	3
8	1
9	1
10	9

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)y_0}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_2)}y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)y_2}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + \frac{(x-x_0)(x-x_1)(x-x_2)y_3}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= \frac{(9.5-8)(9.5-9)(9.5-10)3}{(7-8)(7-9)(7-10)} + \frac{(9.5-7)(9.5-9)(9.5-10)1}{(8-7)(8-9)(8-10)}$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-10)1}{(9-7)(9-8)(9-10)} + \frac{(9.5-7)(9.5-8)(9.5-9)9}{(10-7)(10-8)(10-9)}$$

$$= \frac{-1.125}{-6} + \frac{(-0.625)}{2} + \frac{(-1.875)}{-2} + \frac{16.875}{6}$$

$$= \frac{1.125}{6} - \frac{0.625}{2} + \frac{1.875}{2} + \frac{16.875}{6}$$

$$= \frac{1.125-1.875+5.625+16.875}{6}$$

$$= \frac{21.75}{6} = 3.625$$

Newton's Divided difference table/ formula →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		$\frac{y_1 - y_0}{x_1 - x_0} = \Delta y_0$			
x_1	y_1		$\frac{\Delta y_1 - \Delta y_0}{x_2 - x_0} = \Delta^2 y_0$		
		$\frac{y_2 - y_1}{x_2 - x_1} = \Delta y_1$		$\frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0} = \Delta^3 y_0$	
x_2	y_2		$\frac{\Delta y_2 - \Delta y_1}{x_3 - x_1} = \Delta^2 y_1$		$\frac{\Delta^3 y_1 - \Delta^3 y_0}{x_4 - x_0} = \Delta^4 y_0$
		$\frac{y_3 - y_2}{x_3 - x_2} = \Delta y_2$		$\frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1} = \Delta^3 y_1$	
x_3	y_3		$\frac{\Delta y_3 - \Delta y_2}{x_4 - x_2} = \Delta^2 y_2$		
		$\frac{y_4 - y_3}{x_4 - x_3} = \Delta y_3$			
x_4	y_4				

$$y(x) = y_0 + (x - x_0)\Delta y_0 + (x - x_0)(x - x_1)\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)\Delta^3 y_0 + (x - x_0)(x - x_1)(x - x_2)(x - x_3)\Delta^4 y_0 + \cdots$$

Q→ using Newton divided difference formulla calculate $f(6)$ from the following table →

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
1	1			
		$\frac{5-1}{2-1} = 4$		
2	5		$\frac{0-4}{7-1} = \frac{-2}{3}$	
		$\frac{5-5}{7-2} = 0$		$\frac{-\frac{1}{6}+\frac{2}{3}}{8-1} = \frac{1}{14}$
7	5		$\frac{-1-0}{8-2} = \frac{1}{6}$	
		$\frac{4-5}{8-7} = -1$		
8	4			

$$y(x) = y_0 + (x-x_0)\,\Delta y_0 + (x-x_0)(x-x_1)\,\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\,\Delta^3 y_0$$

$$y(6) = 1 + (6-1)\times 4 + (6-1)(6-2)\times -\frac{2}{3} + (6-1)(6-2)(6-7)\times\frac{1}{14}$$

$$= 1 + 20 - \frac{40}{3} - \frac{20}{14}$$

$$= \frac{42 + 840 - 560 - 60}{42}$$

$$= \frac{262}{42} = 6.2380$$

Q→ If $f(x) = 1/x^2$, find the $f(a,b)$, $f(a,b,c)$, $f(a,b,c,d)$

$f(x) = 1/x^2$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
a	$\frac{1}{a^2}$			
		$\frac{\frac{1}{b^2}-\frac{1}{a^2}}{b-a} = \frac{-(a+b)}{a^2 b^2}$		
b	$\frac{1}{b^2}$		$\frac{\frac{-(b+c)}{b^2c^2} + \frac{(a+b)}{a^2b^2}}{c-a} =$	
		$\frac{\frac{1}{c^2}-\frac{1}{b^2}}{c-b} = \frac{-(b+c)}{b^2c^2}$		
c	$\frac{1}{c^2}$		$\frac{-\frac{(d+c)}{d^2c^2} + \frac{(b+c)}{b^2c^2}}{d-b} =$	
		$\frac{\frac{1}{d^2}-\frac{1}{c^2}}{d-c} = \frac{-(d+c)}{d^2c^2}$		
d	$\frac{1}{d^2}$			

Q→ Find the polynomial from the following table using Newton divided difference formula-

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	15				
		$\frac{-5}{3}$			
-1	10		$\frac{-10}{12} = \frac{-5}{6}$		
		-5		$-\frac{1}{2}$	
0	5		$\frac{13}{6}$		$-\frac{1}{135}$
		$\frac{3}{2}$			
2	8			$\frac{-13}{30}$	
		$-\frac{2}{3}$	$-\frac{13}{30}$		
5	6				

$$y(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 y_0$$

$$= 15 + (x+4)\frac{-5}{3} + (x+4)(x+1)\frac{-5}{6} + (x+4)(x+1)(x-0)\frac{-1}{2} + (x+4)(x+1)(x-0)(x-2)\frac{-1}{135}$$

$$= 15 + \left(\frac{-5x}{3} - \frac{20}{3}\right) + (x^2+x+4x+4)\frac{-5}{6} + (x^2+x+4x+4)(x-0)\frac{-1}{2} + (x^2+x+4x+4)(x-0)(x-2)\frac{-1}{135}$$

$$= 15 - \frac{5x}{3} - \frac{20}{3} + (x^2+5x+4)\frac{-5}{6} + (x^2+5x+4)(x-0)\frac{-1}{2} + (x^2+5x+4)(x-0)(x-2)\frac{-1}{135}$$

$$= 15 - \frac{5x}{3} - \frac{20}{3} - \frac{5x^2}{6} - \frac{25x}{6} - \frac{20}{6} + (x^3+5x^2+4x)\frac{-1}{2} + (x^3+5x^2+4x)(x-2)\frac{-1}{135}$$

$$= 15 - \frac{5x}{3} - \frac{20}{3} - \frac{5x^2}{6} - \frac{25x}{6} - \frac{20}{6} - \frac{x^3}{2} - \frac{5x^2}{2} - \frac{4x}{2} + (x^4 - 2x^3 + 5x^3 - 10x^2 + 4x^2 - 8x)\frac{-1}{135}$$

$$= 15 - \frac{5x}{3} - \frac{20}{3} - \frac{5x^2}{6} - \frac{25x}{6} - \frac{20}{6} - \frac{x^3}{2} - \frac{5x^2}{2} - \frac{4x}{2} - \frac{x^4}{135} + \frac{2x^3}{135} - \frac{5x^3}{135} + \frac{10x^2}{135} - \frac{4x^2}{135} + \frac{8x}{135}$$

$$= \frac{4050 - 450x - 1800 - 225x^2 - 1125x - 900 - 135x^3 - 675x^2 - 540x - 2x^4 + 4x^3 - 10x^3 + 20x^2 - 8x^2 + 16x}{270}$$

$$= \frac{-2x^4 - 141x^3 - 888x^2 - 2099x + 1350}{270}$$

Q→ Find the third divided differentiation with argument 2, 4, 9, 10 of funct^n $f(x) = x^3 - 2x$

x	$f(x)$	$\Delta f x$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	4			
		26		
4	56		15	
		131		1
9	187		23	
		269		
10	980			

$$\Delta^3 f(x) = 1$$

Q→ Prove that

$$\Delta^3_{bcd}\left(\frac{1}{a}\right) = \frac{-1}{abcd}$$

$$f(x) = \frac{1}{x}$$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
a	$\frac{1}{a}$			
		$\frac{\frac{1}{b}-\frac{1}{a}}{b-a} = \frac{-1}{ab}$		
b	$\frac{1}{b}$		$\frac{-1/bc + \frac{1}{ab}}{c-a} = \frac{1}{abc}$	
		$\frac{\frac{1}{c}-\frac{1}{b}}{c-b} = \frac{-1}{bc}$		$\frac{\frac{1}{bcd} - \frac{1}{abc}}{d-a}$
c	$\frac{1}{c}$		$\frac{\frac{-1}{cd} + \frac{1}{bc}}{d-b} = \frac{1}{bcd}$	
		$\frac{\frac{1}{d}-\frac{1}{c}}{d-c} = \frac{-1}{cd}$		
d	$\frac{1}{d}$			

$$= \frac{\frac{(a-d)}{abcd}}{(d-a)} = \frac{(a-d)}{abcd(d-a)} = \frac{-1}{abcd} = R.H.S. \text{ P}$$

Q→ If $f(x) = g(x)\,h(x)$ then prove that

$$f(x_1, x_2) = g(x_1)\,h(x_1, x_2) + g(x_1, x_2)\,h(x_2)$$

$$R.H.S. = g(x_1)\,h(x_1, x_2) + g(x_1, x_2)\,h(x_2)$$

$$= g(x_1)\left[\frac{h(x_2) - h(x_1)}{x_2 - x_1}\right] + \left[\frac{g(x_2) - g(x_1)}{x_2 - x_1}\right] h(x_2)$$

$$= \frac{g(x_1)[h(x_2) - h(x_1)] + [g(x_2) - g(x_1)]\,h(x_2)}{x_2 - x_1}$$

$$= \frac{g(x_1)\cancel{h(x_2)} - g(x_1)h(x_1) + h(x_2)g(x_2) - g(x_1)\cancel{h(x_2)}}{x_2 - x_1}$$

$$= \frac{g(x_2)h(x_2) - g(x_1)h(x_1)}{x_2 - x_1}$$

$$= \frac{f(x_2) - f(x_1)}{x_2 - x_1} = f(x_1, x_2) = L\cdot H\cdot S\cdot$$

Central difference table →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	y_{-2}				
		Δy_{-2}			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
		Δy_{-1}		$\Delta^3 y_{-2}$ above central line	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$ central line
		Δy_0		$\Delta^3 y_{-1}$ below central line	
x_1	y_1		$\Delta^2 y_0$		
		Δy_1			
x_2	y_2				

Gauss forward central diffence formulla →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	y_{-2}				
		Δy_{-2}			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
		Δy_{-1}		$\Delta^3 y_{-2}$	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$
		Δy_0		$\Delta^3 y_{-1}$	
x_1	y_1		$\Delta^2 y_0$		
		Δy_1			
x_2	y_2				

$$f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)\Delta^2 y_{-1}}{2!} + \frac{u(u-1)(u+1)\Delta^3 y_{-1}}{3!} + \frac{u(u-1)(u+1)(u-2)\Delta^4 y_{-2}}{4!} + \cdots$$

This formulla is apply when $0 \le u \le \frac{1}{2}$

Q → Using gauss forward formulla calculate $f(3.75)$.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
2.5	24					
		-19				
3.0	5		30			
		11		-48		
3.5	16		-18		86	
		-7		38		-172
4.0	9		20		-86	
		13		-48		
4.5	22		-28			
		-5				
5.0	7					

$$u = \frac{x - x_0}{h} = \frac{(3.75 - x)}{.5}$$

$$= \frac{(3.75 - 3.5)}{.5} = .5$$

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_1 + \frac{u(u-1)(u+1)}{3!}\Delta^3 y_{-1} + \frac{u(u-1)(u+1)(u}{4!}$$

$$+ \frac{u(u-1)(u+1)(u-2)(u+2)}{5!}\Delta^5 y_{-2}$$

$$= 16 + .5 \times -7 + \frac{.5(.5-1)(-18)}{2} + \frac{.5(.5-1)(.5+1)38}{6} + \frac{.5(.5-1)(.5+1)}{24}$$

$$+ \frac{.5(.5-1)(.5+1)(.5-2)(.5+2)(-172)}{120}$$

$$= 16 - 3.5 + 2.25 - 2.375 + 2015 - 2.015$$

$$= 12.375$$

Q→ Use gauss forward evaluate y_{30}, given that $y_{21} = 18.4708$, $y_{25} = 17.8144$, $y_{29} = 17.1070$, $y_{33} = 16.2432$, $y_{37} = 15.5154$

x	$y(=\times 10^4)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
21	184708				
		-6564			
25	178144		-510		
		-7074		-54	
29	171070		-564		-22
		-7638		-76	
33	162432		-640		
		-8278			
37	155154				

$$u = \frac{x - x_0}{h} = \frac{30 - 29}{4} = 0.25$$

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_1 + \frac{u(u-1)(u+1)}{3!}\Delta^3 y_{-1} + \frac{u(u-1)(u+1)(u}{4!}$$

$$= 171070 + 0.25 \times -7638 + \frac{0.25(0.25-1) - 564}{2} + \frac{0.25(0.25-1)(0.25}{6}$$

$$+ \frac{0.25(0.25-1)(0.25+1)(0.25-2) - 22}{24}$$

$$= 169215 \times \frac{1}{10^4}$$

$$= 16.9215$$

Q- Apply gauss forward central difference formulla to find the value of U_9

$U_0=14$, $U_4=24$, $U_8=32$, $U_{12}=35$, $U_{16}=40$

x	$U=y$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	14				
		10			
4	24		-2		
		8		-3	
8	32		-5		10
		3		7	
12	35		2		
		5			
16	40				

$$U=\frac{x-x_0}{h}=\frac{9-8}{4}=0.25$$

$$y(x)=y_0+U\Delta y_0+\frac{U(U-1)\Delta^2 y_{-1}}{2!}+\frac{U(U-1)(U+1)\Delta^3 y_{-1}}{3!}$$
$$+\frac{U(U-1)(U+1)(U-2)\Delta^4 y}{4!}$$

$$y(9)=32+0.25\times3+\frac{0.25(0.25-1)}{2}\times(-5)+\frac{(0.25)(0.25-1)(0.25+1)7}{6}$$
$$+\frac{0.25(0.25-1)(0.25+1)(0.25-2)\times10}{24}$$

$$=32+0.75+\frac{0.9375}{2}-\frac{1.640625}{6}+\frac{4.1015}{24}$$

$$=768+18+11.25-6.5624+4.1015$$

$$=794$$

$$=32+0.75-0.04-0.54+0.17$$

$$=32.34$$

Gauss backward central difference formulla -

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	y_{-2}				
		Δy_{-2}			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
		Δy_{-1}		$\Delta^3 y_{-2}$	
x_0	y_0		Δy_{-1}		$\Delta^4 y_{-2}$
		Δy_0		Δy_{-1}	
x_1	y_1		Δy_0		
		Δy_1			
x_2	y_2				

$$y(x) = y_0 + u\Delta y_{-1} + \frac{u(u+1)}{2!}\Delta^2 y_{-1} + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-2} + \frac{u(u+1)(u-\ldots}{4!}$$

This formulla is apply when $-\frac{1}{2} \le u \le 0$

Q. Given that

$\sqrt{12500} = 111.803399$

$\sqrt{12510} = 111.848111$

$\sqrt{12520} = 111.892806$

$\sqrt{12530} = 111.937483$

calculate $\sqrt{12516}$.

x	$y = x10^6$	Δy	$\Delta^2 y$	$\Delta^3 y$
12500	111803399			
		44712		
12510	111848111		-17	
		44695		-1
12520	111892806		-18	
		44677		
12530	111937483			

$$u = \frac{x - x_0}{h} = \frac{12516 - 12520}{10} = -0.4$$

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u+1)}{2}\Delta^2 y_{-1} + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-2}$$

$$= 111892806 + (-0.4)(44695) + \frac{(-0.4)(-0.4+1)(-18)}{2} + \frac{(-0.4)(-0.4+1)(-0.4-1)(-1)}{6}$$

$= 111874930.1$

$= \frac{111874930.1}{10^6} = 111.874930$

Q. Using gauss backward interpolation formulla find the p... of year 1936. given that

Year x	population y (in thousand)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1901	12				
		3			
1911	15		2		
		5		0	
1921	20		2		3
		7		3	
1931	27		5		-7
		12		-4	
1941	39		1		
		13			
1951	52				

$$u = \frac{(x - x_0)}{h} = \frac{1936 - 1941}{10}$$

$$= -0.5$$

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u+1)\Delta^2 y_{-1}}{2!} + \frac{u(u+1)(u-1)\Delta^3 y_{-1}}{3!}$$

$$= 39 + (-0.5) \times 12 + \frac{(-0.5)(0.5)}{2} \times 1 + \frac{(-0.5)(0.5)(-1.5) \times -4}{6}$$

$$= 32.625 \approx 33 \text{ thousand}$$

Operators in interpolation →

(1) Central difference operator → δ

$$\delta_{n-\frac{1}{2}} = \delta_n - \delta_{n-1} \quad n = 1, 2, 3, \ldots$$

$$\delta_{1/2} = \delta_1 - \delta_0$$

$$\delta_{3/2} = \delta_2 - \delta_1$$

$$\vdots$$

$$\boxed{\delta_{n-\frac{1}{2}} = \delta_n - \delta_{n-1}}$$

(2) Shift operator → E

$$E f(x) = f(x+h)$$

$$E^2 f(x) = f(x+2h)$$

$$E^3 f(x) = f(x+3h)$$

$$\vdots$$

$$\boxed{E^n f(x) = f(x+nh)}$$

(3) Averaging operator → It is denoted by μ.

$$\boxed{\mu y_x = \frac{1}{2}\left[y_{x+\frac{h}{2}} + y_{x-\frac{h}{2}}\right]}$$

Relationship b/w operators →

(1) $\Delta = E - 1$

(2) $\nabla = 1 - E^{-1}$

(3) $\delta = E^{1/2} + E^{-1/2}$

(4) $\mu = \dfrac{E^{1/2} + E^{-1/2}}{2}$

(5) $E = e^{hD}$

where, e → exponent

h → Diff. b/w two values

$D \rightarrow \left(\frac{d}{dx}\right)$

(6) $\sinh Q = \dfrac{e^{Q} - e^{-Q}}{2}$

(7) $\cosh Q = \dfrac{e^{Q} + e^{-Q}}{2}$

(8) $\tanh Q = \dfrac{e^{Q} - e^{-Q}}{e^{Q} + e^{-Q}}$

(9) $e^{x} = 1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \dfrac{x^4}{4!} - - - -$

(10) $\Delta = E\nabla = \nabla E$

Q→ $\nabla - \Delta = -\nabla\Delta$ Prove that.

$$L.H.S = \nabla - \Delta$$
$$= (1-E^{-1}) - (E-1)$$
$$= \left(1-\frac{1}{E}\right) - (E-1)$$
$$= \frac{(E-1)}{E} - (E-1)$$
$$= \frac{(E-1) - E(E-1)}{E}$$
$$= \frac{E-1-E^2+E}{E} = \frac{-E^2-1+2E}{E}$$
$$= \frac{-(E^2+1-2E)}{E} = \frac{-(E-1)^2}{E}$$
$$= \frac{-(E-1)(E-1)}{E} = \frac{-\Delta(E-1)}{E}$$
$$= \frac{-\Delta \cancel{E}\left(1-\frac{1}{E}\right)}{\cancel{E}} = \boxed{-\Delta(1-E^{-1})}$$
$$= -\Delta\nabla = -\nabla\Delta = R.H.S.$$

Q→ $\Delta + \nabla = \dfrac{\Delta}{\nabla} - \dfrac{\nabla}{\Delta}$

$$R.H.S. = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$$
$$= \frac{(E-1)}{(1-E^{-1})} - \frac{(1-E^{-1})}{(E-1)}$$
$$= \frac{(E-1)}{\left(1-\frac{1}{E}\right)} - \frac{\left(1-\frac{1}{E}\right)}{(E-1)} = \frac{E\cancel{(E-1)}}{\cancel{(E-1)}} - \frac{\cancel{(E-1)}}{E\cancel{(E-1)}}$$

$$= E - \frac{1}{E}$$

$$= E - E^{-1} + 1 - 1$$

$$= (E-1) + (1-E^{-1})$$

$$= \Delta + \nabla = L.H.S.$$