Unit-1

Rules for deciding significant digits→

Rule No.	Definition	Ex.	No. of significant digits
(1.)	All non-zero digits are significant	1234	4
(2.)	Zeroes b/w non-zero digits are significant	1002 100.002	4 6
(3.)	Leading zeroes to the left of the first non-zero digits are not a significant	000200 000010	3 2
(4.)	Trailing zeroes to the right of the decimal point are significant or Trailing zeroes to the right of non-zero digit also to the right of the decimal point are significant	00.0100	3

Ex→

No.	No. of significant digits (figures)	Rule No.
3969	4	1
3060	4	2
39.69	4	1
0.3969	4	3
39.00	4	4
0.00039	2	3
0.00390	3	3, 4
3.0069	5	2

Rounding off→

Rules for rounding off→

(1) Discard all the digits to the right of the n^{th} digit & check the $n+1^{th}$ digit.

Case (i)→ If $n+1^{th}$ digit is >5 then increase the value of n^{th} digit by 1.

12.8378 = 12.84

case (ii)→ If $n+1^{th}$ digit is <5 then n^{th} digit is unchanged.

12.8338 = 12.83

case (iii)→ If $n+1^{th}$ digit is equal to 5 then increase the value of n^{th} digit by 1 if n is odd otherwise unchanged.

12.8354 = 12.84

12.8454 = 12.84

Q→ Round off the following no. to 4th place of decimal point.

No.	Answer	Case No.
6.284937	6.2849	2
7.49956	7.4996	1
1999.456598	1999.4566	1
19.457555	19.4576	3
187.49956	187.4996	1

Q→ Rounding off the following no. to 3 significant figures.

(i) 78.953 = 79.0

(ii) 87.498 = 87.5

(iii) 11.358 = 11.4

(iv) 12.858 = 12.8

(v) 89.895 = 89.9

Truncation→

(1) 0·00894 to 3 decimal places

$= 0·008$

(2) Truncate the no· to 3 significant figures·

$0·0000089456789$

$= 0·00000894$

Representation of floating point no·→

Sign	Exponent	Mantissa
← 1 →	← 7 →	← 24 →

← 32 bit →

If x is a real no· then its floating point representation is $x = f \times 10^{E}$

where f → fraction part (mantissa)

E → Exponent

Rules for converting a no· into the fraction part notation→

(1) 1234

floating point representation $= ·1234 \times 10^{4}$

Rule→ If decimal point is move to the left then power is +ve·

(2) 0·00012345

floating point representation $= 0·12345 \times 10^{-3}$

Rule→ If decimal point is move to the right then power is -ve·

Ex→ (1) 0·00596

$= ·596 \times 10^{-2}$

(2) 65·742

$= ·65742 \times 10^{2}$

(3) −48·6782

$= -·486782 \times 10^{2}$

Normalization of floating point no:-

(1) 1234

Normalization form = $.1234 \times 10^{4}$

Exponent form = .1234 E 4

(2) 596×10^{5}

Normalization form = $.596 \times 10^{8}$

Exponent form = .596 E 8

(3) -65421

Normalization form = $-.65421 \times 10^{5}$

Exponent form = $-.65421$ E 5

(4) .0001234

Normalization form = $.1234 \times 10^{-3}$

Exponent form = .1234 E -3

(5) -0.00986

Normalization form = $-.986 \times 10^{-2}$

Exponent form = $-.986$ E -2

(6) 12345

Normalization form = $.12345 \times 10^{5}$

Exponent form = .12345 E 5

Fraction part operations:-

(1) Addition:-

Algorithm:-

Step (1) → Set: $E_z = \max(E_x, E_y)$. Suppose E_x is max

$$E_z = E_x$$

Step (2) → Shift right: If E_x is max then shift f_y by $(E_x - E_y)$ places

If E_y is max then shift f_x by $(E_y - E_x)$ places

Step (3) → Add the fraction part:

$$f_z = f_x + f_y$$

Step(4) → Result →

$z = f_z E_z$

Step(5) → Normalized z.

Q → $x = 0.964572 E2$

$y = 0.506351 E5$

$f_x = 0.964572$, $E_x = E2$

$f_y = 0.506351$, $E_y = E5$

$E_z = \max(E_x, E_y)$

$E_z = \max(E2, E5)$

$E_z = E_y = E5$

Shift f_x by $(E_y - E_x)$ places

$f_x = 0.964572$ shift to right

$f_x = 0.000964$

$f_z = f_x + f_y$

$= 0.000964 + 0.506351$

$= 0.507315$

$z = f_z E_z$

$= 0.507315 E5$

Q → $x = 98.67402 \times 10^4$

$y = 0.0009642892 \times 10^{10}$

$x = 0.9867402 \times 10^6$

$= 0.9867402 \; E6$

$y = 0.9642892 \times 10^7$

$= 0.9642892 \; E7$

$f_x = 0.9642892$, $E_x = E6$

$f_y = 0.9642892$, $E_y = E7$

$E_z = \max(E_x, E_y)$

$E_z = \max(E6, E7)$

Shift f_x by $(E_y - E_x)$ places

$f_x = 0.9867482$ shift to right

$f_x = 0.0986748$

$f_z = f_x + f_y$

$= 0.0986748 + 0.9642892$

$= 1.0629640$

$z = f_z E_z$

$= 1.0629640 E7$

$z = 0.1062964 E8$

(2) Subtraction→

Algorithm→

Step (1)→ Set→ $E_z = \max(E_x, E_y)$, suppose E_x is max

$E_z = E_x$

Step (2)→ Shift right→ If E_x is max then shift f_y by $(E_x - E_y)$ places.

If E_y is max then shift f_x by $(E_y - E_x)$ places.

Step (3)→ ~~Add~~ Sub the fraction part→

$f_z = f_x - f_y$

Step (4)→ Result→

$z = f_z E_z$

Step (5)→ Normalized z.

Q→ Subtract 0.994576 E-3 from 0.999658 E-3

Let $x = 0.999658$ E-3

$y = 0.994576$ E-3

$f_x = 0.999658$, $E_x = E-3$

$f_y = 0.994576$, $E_y = E-3$

$E_z = \max(E_x, E_y)$

$E_z = E_x = E-3$

Shift right fy by $(Ex - Ey)$ places

$fy = 0.994576$

$fz = fx - fy$

$= 0.999658 - 0.994576$

$= 0.005082$

$z = fz\,Ez$

$= 0.005082\,E{-3}$

$z = 0.508200\,E{-5}$

Q→ Subtract $22.98252\,E{-5}$ from $99.28753\,E0$

~~$x = 0.22982$~~

Let $x = 99.28753\,E0 = .9928753\,E2$

$y = 22.98252\,E{-5} = .2298252\,E{-3}$

$fx = .9928753,\ Ex = E2$

$fy = .2298252,\ Ey = E{-3}$

$Ez = \text{Max}(Ex, Ey)$

$= \max(E2, E{-3})$

$Ez = Ex = E2$

Shift right ~~by~~ fy by $(Ex - Ey)$ places

$fy = .2298252$

$= 0.0000022$

$fz = fx - fy$

$= 0.9928753 - 0.0000022$

$= 0.9928731$

$z = fz\,Ez$

$= 0.9928731\,E2$

(3) Multiplication→

Algorithms→

Step (1) → Add the exponent→

$Ez = Ex + Ey$

Step (2) → Multiply the fraction part →

$$f_z = f_x * f_y$$

Step (3) → Result →

$$z = f_z E_z$$

Step (4) → Normalized z.

Q → $.5543\,E12$ & $.4111\,E-15$

let $x = .5543\,E12$

$y = .4111\,E-15$

$f_x = .5543, \quad E_x = E12$

$f_y = .4111, \quad E_y = E-15$

$E_z = E_x + E_y$

$= 12 - 15 = E-3$

$f_z = f_x * f_y$

$= .5543 * .4111$

$= 0.22787273 = 0.2279$

$z = f_z E_z$

$= 0.2279\,E-3$

(4) Division →

Algorithm →

Step (1) → Subtract the exponent →

$$E_z = E_x - E_y$$

Step (2) → Divide the fraction part →

$$f_z = f_x / f_y$$

Step (3) → $z = f_z E_z$

Step (4) → Normalize z.

Q → $.9998\,E1 \div 1000\,E-99$

$x = .9998\,E1$

$y = 1000\,E-99 = .1000\,E-95$

$f_x = .9998, \quad E_x = E1$

$\lambda y = \cdot 1000, \; E_y = E - 95$

$E_z = E_x - E_y$

$= 1 + 95 = 96$

$\lambda z = \lambda x / \lambda y$

$= \dfrac{\cdot 9998}{\cdot 1000}$

$= 9.9980$

$z = 9.9980 E96$

$z = 0.9998 E97$

Errors in Numerical computation →

(1) Inherit error

(2) Rounding off error →

$x = \dfrac{1}{3}$

$\bar{x} = 0.333$

$R.O.E. = x - \bar{x}$

(3) Truncation error →

$T.E. = x - \bar{x}$

(4) Absolute, Relative & Percentage error →

$A.E. (E_a) = |x - \bar{x}|$

$R.E. (E_r) = \left| \dfrac{x - \bar{x}}{x} \right| = \left| \dfrac{E_a}{x} \right|$

$P.E. (E_p) = 100 \cdot E_r$

$= 100 * \left| \dfrac{x - \bar{x}}{x} \right|$

Q → Round off the no. 865250 & 37.46235 to 4 significant digits? find E_a, E_r, E_p?

Let $x = 865250$

$= 8652$

$E_a = |x - \bar{x}|$

$= |865250 - 865200|$

$= 50$

$E_r = \left|\frac{X - \bar{X}}{X}\right|$

$= \left|\frac{865250 - 865200}{865250}\right|$

$= \cdot 0001$

$E_p = 100 \cdot E_r$

$= 100 * 0 \cdot 0001$

$= 0 \cdot 0001 * 10^2$

or

$E_r = \left|\frac{X - \bar{X}}{X}\right|$

$= \left|\frac{865250 - 865200}{865250}\right|$

$= 5 \cdot 778676683 * 10^{-5}$

$= 5 \cdot 778 * 10^{-5}$

$= \cdot 5778 * 10^{-4}$

$E_p = 100 \cdot E_r$

$= 100 * \cdot 5778 * 10^{-4}$

$= \cdot 5778 * 10^{-2}$

Let $X = 37 \cdot 46235$

$\bar{X} = 37 \cdot 46000$

$E_a = |X - \bar{X}|$

$= |37 \cdot 46235 - 37 \cdot 46000|$

$= \cdot 00235$

$E_r = \left|\frac{E_a}{X}\right|$

$= \left|\frac{\cdot 00235}{37 \cdot 46235}\right|$

$= 6 \cdot 272964723 * 10^{-5}$

$= 6.273 * 10^{-5}$

$= .6273 * 10^{-4}$

$E_p = 100 * E_r$

$= 100 * .6273 * 10^{-4}$

$= .6273 * 10^{-2}$

Overflow and underflow condition →

$-99 \leq E \leq 99$

underflow — overflow

Q → Multiply the numbers

$x = 0.350000 E 40$

$y = 0.500000 E 70$

$fx = 0.350000$, $Ex = E40$

$fy = 0.500000$, $Ey = E70$

$Ez = Ex + Ey$

$= 40 + 70 \cancel{= 130}$

$= E110$

$fz = fx * fy$

$fz = 0.350000 * 0.500000$

$= 0.175000$

$z = fz\, Ez$

$= 0.175000 E 110$

$\cancel{= 0.175}$

Because the range of E is > 99. So, result is overflow.

Sol^n of algebric and transcendental eq^n →

Interactive methods for finding the root of eq^n →

(1) Bisection method →

working procedure -

Let $f(x) = ax^2 + bx + c$ — ①

Now we make the table

x	0	1	2	3
f(x)	+ve	+ve	+ve	-ve

Let $x_0 = -ve \Rightarrow f(x_0)$

$x_1 = +ve \Rightarrow f(x_1)$

Now, we calculate the value of

$$x_2 = \frac{(x_0 + x_1)}{2}$$

$$f(x_2) = ?$$

Case(1) → if $(f(x_2) < 0)$

$x_0 = x_2$;

$f(x_0) = f(x_2)$

Case(2) → else if $(f(x_2) > 0)$

$x_1 = x_2$

$f(x_1) = f(x_2)$

Q → Find the root of an Eq^n

$x^3 - 4x - 9 = 0$ upto 4

decimal places by using bisection method.

Given $f(x) = x^3 - 4x - 9 = 0$

Now we make the table

x	0	1	2	3
fx	-9.0000	-12.0000	-9.0000	6.0000

Let $x_0 = 2$, $\Rightarrow f(x_0) = -9.0000$

$x_1 = 3 \Rightarrow f(x_1) = 6.0000$

Now we calculate the value at

$$x_2 = \frac{(x_0 + x_1)}{2} = \frac{2+3}{2} = 2.5000$$

$f(x_2) = -3.3750$

Now we make the table

x_0 (a)	$f(x_0)$	x_1 (b)	$f(x_1)$	x_2 (c)	$f(x_2)$
2.0000	-9.0000	3	6.0000	2.5000	-3.3750
2.5000	-3.3750	3	6.0000	2.7500	0.7969
2.5000	-3.3750	2.7500	0.7969	2.6250	-1.4121
2.6250	-1.4121	2.7500	0.7969	2.6875	-0.3391
2.6875	-0.3391	2.7500	0.7969	2.7188	0.2218
2.6875	-0.3391	2.7188	0.2218	2.7032	-0.0597
.7032	-0.0597	2.7188	0.2218	2.7110	0.0806
.7032	-0.0597	2.7110	0.0806	2.7071	0.0103

2.7032	-0.0597	2.7071	0.0103	2.7052	-0.0239
2.7052	-0.0239	2.7071	0.0103	2.7062	-0.0059
2.7062	-0.0059	2.7071	0.0103	2.7067	0.0031
2.7062	-0.0059	2.7067	0.0031	2.7065	-0.0005
2.7065	-0.0005	2.7067	0.0031	2.7066	0.0013
2.7066	0.0013	2.7067	0.0031	2.7067	0.0031
2.7067	0.0031	2.7067	0.0031	2.7067	0.0031

Since, the value of last two approximation are same.
Hence, the root of the eqn is 2.7066.

$f(x) = 0.0013$ at $x = 2.7066$

Q→ find the root of the eqn

$2x - \log_{10} x - 7$ upto 4 decimal places.

Given $f(x) = 2x - \log_{10} x - 7$

Now we make the table.

In (log) value of x is started from 1.

x	1	2	3	4
$f(x)$	-5.0000	-3.3010	-1.4771	0.3979

$x_0 = 3 \quad f(x_0) = -1.4771$

$x_1 = 4 \quad f(x_1) = 0.3979$

$$x_2 = \frac{(x_0 + x_1)}{2} = 3.5000$$

$f(x_2) = -0.5441$

Now we make the table

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$
3.0000	-1.4771	4.0000	0.3979	3.5000	-0.5441
3.5000	-0.5441	4.0000	0.3979	3.7500	-0.0704
3.7500	-0.0740	4.0000	0.3979	3.8750	0.1617
3.7500	-0.0740	3.8750	0.1617	3.8125	0.0438
3.7500	-0.0740	3.8125	0.0438	3.7813	-0.0150
3.7813	-0.0150	3.8125	0.0438	3.7969	0.0144
3.7813	-0.0150	3.7969	0.0144	3.7891	-0.0003
3.7891	-0.0003	3.7969	0.0144	3.7930	0.0070
3.7891	-0.0003	3.7930	0.0070	3.7911	0.0034

3·7891	−0·0003	3·7911	0·0034	3·7901	0·0015
3·7891	−0·0003	3·7901	0·0015	3·7896	0·0006
3·7891	−0·0003	3·7896	0·0006	3·7894	0·0002
3·7891	−0·0003	3·7894	0·0002	3·7893	0·0000

$f(x) = 0·0000$ at $x = 3·7893$

(2) Iteration method→

Working procedure–

$$f(x) = ax^2 + bx + c \quad —①$$

Now we make the table

x	0	1	2
$f(x)$	−ve	−ve	+ve

Let $x_0 = -ve$, $f(x_0) = -ve$

$x_1 = +ve$, $f(x_1) = +ve$

$$x_2 = \frac{(x_0 + x_1)}{2}$$

write the given eqn $f(x)$ into the,

$$\boxed{x = \phi(x)}$$

Set, the value of initial approximation

$$\boxed{x_0 = x_2}$$

Apply the iteration method

$$x_{i+1} = \phi(x_i) \quad , \quad i = 0, 1, 2, 3, ---$$

$$x_1 = \phi(x_0)$$
$$x_2 = \phi(x_1)$$
$$x_3 = \phi(x_2)$$

$$\boxed{x_{i+1} = \phi(x_{i+1})}$$

Q→ Find the root of eqn $\cos x = 3x - 1$ upto 4 decimal places.

Given $f(x) = \cos x - 3x + 1 \quad —①$

x	0	1
$f(x)$	2·0000	−1·4597

$$x_2 = \frac{(x_0 + x_1)}{2}$$

$$= \frac{0+1}{2} = 0·5000$$

Now, we write the given Eq^n ① into the

$$\boxed{x = \phi(x)}$$

$$3x = \cos x + 1$$

$$x = \frac{(\cos x + 1)}{3}$$

$$\boxed{\phi(x) = \frac{(\cos x + 1)}{3}}$$ **

Set. $x_0 = x_2$

$x_0 = 0.5000$

Apply the iteration method

$$\boxed{x_{i+1} = \phi(x_i)} \quad i = 0, 1, 2, 3, ---$$

$x_1 = \phi(x_0)$

$x_1 = 0.6259$

$x_2 = \phi(x_1) = 0.6035$

$x_3 = \phi(x_2) = .6078$

$x_4 = \phi(x_3) = .6070$

$x_5 = \phi(x_4) = .6071$

$x_6 = \phi(x_5) = .6071$

Since, the value of $funct^n$ is 0.0000 at $x = 0.6071$. Hence, the root of the eq^n is 0.6071.

$f(x) = 0.0000$ at $x = .6071$

Q→ $2x - \log_{10} x = 7$

Given, $f(x) = 2x - \log_{10} x - 7$ ——①

x	1	2	3	4
$f(x)$	-5.0000	-3.3010	-1.4771	0.3979

$$x_2 = \frac{(x_0 + x_1)}{2}$$

$$= \frac{3+4}{2} = 3.5000$$

Now, we write the given Eq^n ① into the

$$\boxed{x = \phi(x)}$$

$$2x = 7 + \log_{10} x$$

$$x = (7 + \log_{10} x)/2$$

$$\boxed{\phi(x) = (7 + \log_{10} x)/2}$$ **

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Let $x_0 = x_0$

$x_0 = 3.5000$

Apply the iteration method

$$\boxed{x_{n+1} = \phi(x_n)} \quad n = 0, 1, 2$$

$x_1 = \phi(x_0)$

$x_1 = 3.7728$

$x_2 = \phi(x_1) = 3.7844$

$x_3 = \phi(x_2) = 3.7857$

$x_4 = \phi(x_3) = 3.7851$

$x_5 = \phi(x_4) = 3.7853$

Since, the value of [illegible] is same at $x = 3.7853$. Hence, the root of the eq^n is 3.7853

$f(x) = x$ [illegible] at $x = 3.7853$

Differentiate formulae -

(1) $\frac{d}{dx}(\text{constant}) = 0$

(2) $\frac{d}{dx}(x^n) = nx^{n-1}$

(3) $\frac{d}{dx} e^x = e^x$

(4) $\frac{d}{dx} \log_a x = \frac{1}{x} \log_a e$

(5) $\frac{d}{dx} \log_e x = \frac{1}{x}$

(6) $\frac{d}{dx} a^x = a^x \log_e a$

(7) $\frac{d}{dx} \sin x = \cos x$

(8) $\frac{d}{dx} \cos x = -\sin x$

(9) $\frac{d}{dx} \tan x = \sec^2 x$

(10) $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

(11) $\frac{d}{dx} \sec x = \sec x \tan x$

(12) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$

(13) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

(14) $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

(15) $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

(16) $\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$

(17) $\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$

(18) $\frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$

(19) Product rule →

$$\frac{d}{dx}(I \cdot II) = I \cdot \frac{d}{dx}(II) + II \frac{d}{dx}(I)$$

(20) Divide rule →

$$\frac{d}{dx}\left(\frac{N}{D}\right) = \frac{N\frac{d}{dx}(D) - D\frac{d}{dx}(N)}{D^2}$$

Newton's Raphson Method / Newton's iteration method →

Derivation →

Let x_0 be an approximate root of the eqn $f(x)$ and x_1 is the exact root of the eqn $f(x)$.

$$\boxed{x_1 = x_0 + h} \quad \text{—— (1)}$$

where h be a smaller quantity.

$$f(x_1) = 0$$

from eqn (1)

$$f(x_0+h) = 0 \text{ [expand by Taylor's series]}$$

$$f(x_0) + h f'(x_0) + \frac{h^2}{2} f''(x_0) + \cdots = 0$$

since h is a smaller quantity so we neglecting higher

$$h f'(x_0) = -f(x_0)$$

$$\boxed{h = \frac{-f(x_0)}{f'(x_0)}}$$

Put the value of h in Eq^n ①

$$x_1 = x_0 + h$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

similarly,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\vdots$$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}} \quad \text{where } i = 0, 1, 2, 3, ---$$

working procedure →

Given, $\boxed{f(x) = ax^2 + bx + c}$ — ①

Now, we make the table

x	0	1	2
$f(x)$	+ve	+ve	−ve

Calculate the value of first approximation –

$$|f(x=1)| < |f(x=2)|$$

set $\boxed{x_0 = 1}$ — ②

Now, we find $f'(x)$

$\boxed{f'(x) =}$ — ③

Apply the Newton's iteration method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \text{where } i = 0, 1, 2, 3, ---$$

Q→ Find the root of the Eqⁿ $x \log_{10} x = 1.2$ upto 4 decimal places.

Given $\boxed{f(x) = x\log_{10} x - 1.2}$ —①

Now, we make the table

x	1	2	3
$f(x)$	-1.2000	-0.5979	0.2314

set x_0 →

$$|f(x=2)| > |f(x=3)|$$

$\boxed{x_0 = 3}$ —②

Calculate $f'(x)$

$$f'(x) = \frac{d}{dx}(f(x))$$

$$= \frac{d}{dx}(x\log_{10} x - 1.2)$$

$$= x * \frac{1}{x}\log_{10} e + \log_{10} x - 0$$

$\boxed{f'(x) = \log_{10} e + \log_{10} x}$ —③

Apply the Newton's iteration method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \text{where } i = 0, 1, 2, ---$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2.7462$$

$$x_2 = 2.7406$$

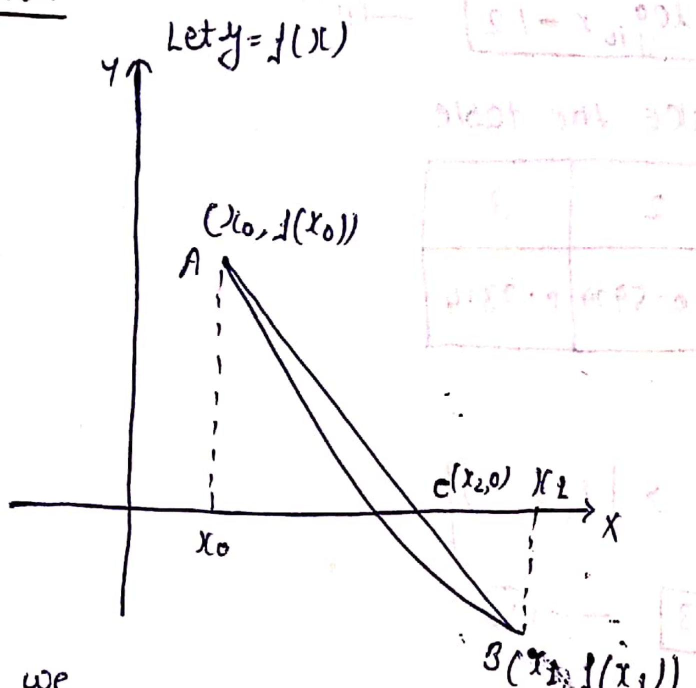
$$x_3 = 2.7406$$

Since, the value of x_2 & x_3 are same. Hence, the value of root is 2.7406.

Regula-falsi method (Method of false position) →

Derivation →

Let $y = f(x)$



Let ~~us~~ we consider two point x_0 & x_1 such that $f(x_0)$ and $f(x_1)$ are of opposite sign. This indicate a root is lying b/w x_0 & x_1.

we know that the eqⁿ of the line joinning the points AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

The line AB passes through point $C(x_2, 0)$

$$\Rightarrow \left.\begin{matrix} x = x_2 \\ y = 0 \end{matrix}\right\}$$

$$0 - f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_0)$$

$$-f(x_0)(x_1 - x_0) = [f(x_1) - f(x_0)](x_2 - x_0)$$

$$-f(x_0)x_1 + x_0 f(x_0) = [f(x_1) - f(x_0)]x_2 - x_0[f(x_1) - f(x_0)]$$

~~$[f(x_1)$~~

$$-f(x_0)x_1 + x_0 f(x_0) + x_0[f(x_1) - f(x_0)] = [f(x_1) - f(x_0)]x_2$$

$$-f(x_0)x_1 + x_0 \cancel{f(x_0)} + x_0 f(x_1) - x_0 \cancel{f(x_0)} = [f(x_1) - f(x_0)]x_2$$

$$x_0 f(x_1) - x_1 f(x_0) = [f(x_1) - f(x_0)]x_2$$

$$\boxed{x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}}$$

In general,

$$\boxed{x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}} \quad i = 1, 2, 3, \ldots$$

Working Procedure→

Given → $\boxed{f(x) = ax^2 + bx + c}$ — ①

Now, we make the table

x	0	1	2
$f(x)$	$-$ve	$-$ve	+ve

Let $x_0 = 1 \Rightarrow f(x_0) = +ve$

$x_1 = 2 \Rightarrow f(x_1) = -ve$

Now we calculate x_2

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$\boxed{f(x_2) = \quad}$

Case 1 → If $f(x_2) f(x_1) < 0$

$x_0 = x_2$

$f(x_0) = f(x_2)$

Case 2 → else if $f(x_2) f(x_0) < 0$

$x_1 = x_2$

$f(x_1) = f(x_2)$

Q→ $f(x) = xe^x - 3$ upto 4 decimal places.

Given, $\boxed{f(x) = xe^x - 3}$ — ①

Now, we make the table

x	0	1	2
$f(x)$	-3.0000	-0.2817	11.7781

Ist approximation→

$x_0 = 1 \Rightarrow f(x_0) = -0.2817$

$x_1 = 2 \Rightarrow f(x_1) = 11.7781$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$x_2 = 1.0234$

$f(x_2) \doteq -0.1522$

$f(x_2) f(x_1) < 0$

Set

$x_0 = x_2 = 1.0234$

$f(x_0) = f(x_2) = -0.1522$

IInd approximation →

$x_0 = 1.0234 \Rightarrow f(x_0) = -0.1522$

$x_1 = 2 \Rightarrow f(x_1) = 11.7781$

$x_2 = 1.0359$

$f(x_2) = -0.0812$

IIIrd approximation →

$x_0 = 1.0359, \; f(x_0) = -0.0812$

$x_1 = 2, \; f(x_1) = 11.7781$

$x_2 = 1.0425$

$f(x_2) = -0.0432$

IVth approximation

$x_0 = 1.0425, \; f(x_0) = -0.0432$

$x_1 = 2, \; f(x_1) = 11.7781$

$x_2 = 1.0460$

$f(x_2) = -0.0228$

Vth approximation

$x_0 = 1.0460, \; f(x_0) = -0.0228$

$x_1 = 2, \; f(x_1) = 11.7781$

$x_2 = 1.0479$

$f(x_2) = -0.01217$

Q→ Find the zeroes of a transcedental eq^n^ $x^2 + 4\sin x = 0$ using Regula falsi method upto 4 decimal places.

x	0	1	2	3	4	5	6
$f(x)$	0.0000	4.3659	7.6372	9.5645	12.9728	21.1643	34.8823

x	-1	-2
$f(x)$	-2.3659	0.3628

Given $\boxed{f(x) = x^2 + 4\sin x}$ — ①

<u>Ist approximation→</u>

$x_0 = -1,\ f(x_0) = -2.3659$

$x_1 = -2,\ f(x_1) = 0.3628$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\boxed{\begin{aligned} x_2 &= -1.8670 \\ f(x_2) &= -0.3401 \end{aligned}}$$

$\Rightarrow f(x_2)f(x_1) < 0$

set, $x_0 = x_2 = -1.8670$

$f(x_0) = f(x_2) = -0.3401$

IInd approximation→

$x_0 = -1.8670,\ f(x_0) = -0.3401$

$x_1 = -2,\ f(x_1) = 0.3628$

$$\boxed{\begin{aligned} x_2 &= -1.9314 \\ f(x_2) &= -0.0124 \end{aligned}}$$

$\Rightarrow f(x_2)f(x_1) < 0$

set, $x_0 = -1.9314$

$f(x_0) = -0.0124$

IIIrd approximation→

$x_0 = -1.9314,\ f(x_0) = -0.0124$

$x_1 = -2,\ f(x_1) = 0.3628$

$$\boxed{\begin{aligned} x_2 &= -1.9337 \\ f(x_2) &= -0.0003 \end{aligned}}$$

$\Rightarrow f(x_2)f(x_1) < 0$

set, $x_0 = -1.9337$

IV th approximation→

$x_0 = -1.9337, \quad f(x_0) = -0.0003$

$x_1 = -2, \quad f(x_1) = 0.3628$

$$\boxed{\begin{array}{l} x_2 = -1.9338 \\ f(x_2) = 0.0002 \end{array}}$$

$\Rightarrow f(x_2)\,f(x_0) < 0$

set, $x_1 = -1.9338$

$f(x_1) = 0.0002$

V th approximation→

$x_0 = -1.9337, \quad f(x_0) = -0.0003$

$x_1 = -1.9338, \quad f(x_1) = 0.0002$

$x_2 = -1.9338$

$f(x_2) = 0.0002$

Since, the value of last two approximation is -1.9338.

Hence, $f(x) = 0.0002$ at $x = -1.9338$

Secant Method→

working procedure→

$f(x) = ax^2 + bx + c$

x	0	1
$f(x)$	+ve	+ve

Ist approximation→

$x_0 = 0, \quad f(x_0) = +ve$

$x_1 = 1, \quad f(x_1) = +ve$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$f(x_2) = +ve$

IInd approximation→

$x_0 = x_1, \quad f(x_0) = f(x_1)$

$x_1 = x_2, \quad f(x_1) = f(x_2)$

$x_2 = ?$

$f(x_2) = ?$

Q→ $xe^x - 3$

$f(x) = xe^x - 3$ — ①

Now, we make the table

x	0	1
$f(x)$	-3.000	-0.282

Ist approximation→

$x_0 = 0,\ f(x_0) = -3.000$

$x_1 = 1,\ f(x_1) = -0.282$

$x_2 = 1.104$

$f(x_2) = 0.330$

IInd approximation→

$x_0 = 1,\ f(x_0) = -0.282$

$x_1 = 1.104,\ f(x_1) = 0.330$

$x_2 = 1.048$

$f(x_2) = -0.011$

IIIrd approximation→

$x_0 = 1.104,\ f(x_0) = 0.330$

$x_1 = 1.048,\ f(x_1) = -0.011$

$x_2 = 1.050$

$f(x_2) = 0.001$

IVth approximation→

$x_0 = 1.048,\ f(x_0) = -0.011$

$x_1 = 1.050,\ f(x_1) = 0.001$

$x_2 = 1.050$

$f(x_2) = 0.001$

Since, the value of last two approximation is 1.050.

Hence,

$f(x) = 0.001$ at $x = 1.050$

Q→ $\tan x + \tanh x = 0$

$f(x) = \tan x + \tanh x$ — ②

Now, we make the table

x	0	1
fx	0.0000	0.7790

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Ist approximation→

$x_0 = 0, \; f(x_0) = 0.0000$

$x_1 = 1, \; f(x_1) = 0.7790$

$x_2 = 0.0000$

$f(x_2) = 0.0000$

IInd approximation→

$x_0 = 1, \; f(x_0) = 0.7790$

$x_1 = 0.0000, \; f(x_1) = 0.0000$

$x_2 = 0.0000$

$f(x_2) = 0.0000$

Since, the value of last two approximation is 0.0000.
Hence, $f(x) = 0.0000$ at $x = 0.0000$

Approximate Numbers→ There are two types of numbers exact and approximate. Exact numbers are 2, 4, 9, 13, 7/2, 6.45, ... etc. But there are numbers such as $4/3 = (1.3333...$, $\sqrt{2} = (1.414213---)$ and $\pi = (3.141592---)$ which can not be expressed by a finite no. of digits.

Significant figures→ The digits used to express a no. are called significant digits.

Rounding off→ There are numbers with large numbers of digits
ex. - $22/7 = 3.142857143$. In practise, it is desirable to limit such numbers to a manageable number of digits such as 3.14 and 3.143. This process of dropping unwanted digits is called rounding off.

Q→ Find the value of $\sqrt{12}$.

Let $x = \sqrt{12}$

squaring on both sides

$$x^2 = 12$$

$$x^2 - 12 = 0$$

$$\boxed{f(x) = x^2 - 12} \quad \text{— ①}$$

Now, we make the table

x	0	1	2	3	4
$f(x)$	-12.0000	-11.0000	-8.0000	-3.0000	4.0000

set $x_0 \Rightarrow$

$$|f(x=3)| \lessgtr |f(x=4)|$$

$$\boxed{x_0 = 3} \quad \text{— ②}$$

Calculate $f'(x)$

$$f'(x) = \frac{d}{dx}(f(x))$$

$$= \frac{d}{dx}(x^2 - 12)$$

$$= 2x - 0 = 2x$$

$$\boxed{f'(x) = 2x} \quad \text{— ③}$$

Apply the Newton's iteration method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad i = 0, 1, 2, 3, 4, \ldots$$

$x_1 = 3.5000$

$x_2 = 3.4643$

$x_3 = 3.4641$

$x_4 = 3.4641$

Since, the value of x_3 & x_4 are same. Hence, the value of root is 3.4641.

$$\boxed{f(x) = 0.0000 \text{ at } x = 3.4641}$$

Q→ find the real root of the Eq^n.

$$x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} - \frac{x^{11}}{1320} + \cdots = 0.443$$

first, we convert given series into $x = \phi(x)$ form

$$x = \frac{x^3}{3} - \frac{x^5}{10} + \frac{x^7}{42} - \frac{x^9}{216} + \frac{x^{11}}{1320} - \cdots + 0.443$$

compare with $x = \phi(x)$

$$\boxed{\phi(x) = \frac{x^3}{3} - \frac{x^5}{10} + \frac{x^7}{42} - \frac{x^9}{216} + \frac{x^{11}}{1320} - \cdots + 0.443}$$

Set $x_0 = 1$

Now, we apply iteration method

$$\boxed{x_{i+1} = \phi(x_i)}, \quad i = 0, 1, 2, \cdots$$

$x_1 = 0.696$

$x_2 = 0.541$

$x_3 = 0.491$

$x_4 = 0.480$

$x_5 = 0.477$

$x_6 = 0.477$

Since, the value of $funct^n$ is 0.000 at $x = 0.477$. Hence, the root of the Eq^n is 0.477.

$$f(x) = 0.000 \text{ at } x = 0.477.$$

Convergence rate of a Newton Raphson method→

Let x_n be the approximate root of the Eq^n $f(x) = 0$ and α be the exact root of the Eq^n $f(x) = 0$

so that
$$\left.\begin{aligned} x_n &= \alpha + \varepsilon_n \\ x_{n+1} &= \alpha + \varepsilon_{n+1} \end{aligned}\right\} \text{ where } \varepsilon \text{ be the smaller quantity.}$$

we know that the formulla of Newton Raphson is

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}} \quad \text{—①}$$

Put the value of x_n & x_{n+1} in Eq^n ①

$$\cancel{\alpha} + \varepsilon_{n+1} = (\cancel{\alpha} + \varepsilon_n) - \frac{f(\alpha + \varepsilon_n)}{f'(\alpha + \varepsilon_n)}$$

$$\varepsilon_{n+1} = \varepsilon_n - \frac{f(\alpha + \varepsilon_n)}{f'(\alpha + \varepsilon_n)}$$

Expand $f(\alpha+\varepsilon_n)$ & $f'(\alpha+\varepsilon_n)$ by Taylor's series.

$$\varepsilon_{n+1} = \varepsilon_n - \frac{[f(\alpha) + \varepsilon_n f'(\alpha) + \frac{\varepsilon_n^2}{2} f''(\alpha) + \cdots]}{[f'(\alpha) + \varepsilon_n f''(\alpha) + \frac{\varepsilon_n^2}{2} f'''(\alpha) + \cdots]}$$

$$\Rightarrow \boxed{f(\alpha)=0}$$

$$\varepsilon_{n+1} = \varepsilon_n - \frac{[\varepsilon_n f'(\alpha) + \frac{\varepsilon_n^2}{2} f''(\alpha) + \cdots]}{[f'(\alpha) + \varepsilon_n f''(\alpha) + \frac{\varepsilon_n^2}{2} f''(\alpha) + \cdots]}$$

Since ε_n is a smaller quantity. So, we neglecting higher terms of ε_n.

$$\varepsilon_{n+1} = \varepsilon_n - \frac{[\varepsilon_n f'(\alpha) + \frac{\varepsilon_n^2}{2} f''(\alpha)]}{[f'(\alpha) + \varepsilon_n f''(\alpha)]}$$

$$\varepsilon_{n+1} = \frac{\cancel{\varepsilon_n f'(\alpha)} + \varepsilon_n^2 f''(\alpha) - \cancel{\varepsilon_n f'(\alpha)} - \frac{\varepsilon_n^2}{2} f''(\alpha)}{[f'(\alpha) + \varepsilon_n f''(\alpha)]}$$

$$\varepsilon_{n+1} = \frac{\varepsilon_n^2 f''(\alpha) - \frac{\varepsilon_n^2}{2} f''(\alpha)}{f'(\alpha) + \varepsilon_n f''(\alpha)}$$

$$\varepsilon_{n+1} = \frac{\frac{1}{2}\varepsilon_n^2 f''(\alpha)}{f'(\alpha) + \varepsilon_n f''(\alpha)}$$

$$\Rightarrow \boxed{\varepsilon_{n+1} \propto \varepsilon_n^2}$$

This shows that the Newton Raphson method has a quadratic convergence.

Q→ Determine the value of p & q. Show that rate of convergence of iterative method $x_{n+1} = p x_n + \frac{q}{x_n^2} N$. For computing $N^{1/3}$ becomes as high as possible.

Let $x = N^{1/3}$

$x^3 = N$

$$\boxed{f(x) = x^3 - N} \quad \text{—} (1)$$

Let α be the root of the Eq^n. then

$f(\alpha) = 0$

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$\boxed{\alpha^3 - N = 0} \Rightarrow \boxed{N = \alpha^3}$

By using convergence rate

$$\left.\begin{aligned} x_n &= \alpha + \varepsilon_n \\ x_{n+1} &= \alpha + \varepsilon_{n+1} \end{aligned}\right\} \quad -② $$

Put the value of Eq^n ② in given funct^n

$$\alpha + \varepsilon_{n+1} = p(\alpha + \varepsilon_n) + \frac{q}{(\alpha+\varepsilon_n)^2}\alpha^3$$

$$\alpha + \varepsilon_{n+1} = p\alpha + p\varepsilon_n + \frac{q\alpha^3}{\alpha^2\left(1+\frac{\varepsilon_n}{\alpha}\right)^2}$$

$$\alpha + \varepsilon_{n+1} = p\alpha + p\varepsilon_n + \frac{q\alpha}{\left(1+\frac{\varepsilon_n}{\alpha}\right)^2}$$

$$\varepsilon_{n+1} = p\alpha - \alpha + p\varepsilon_n + q\alpha\left(1+\frac{\varepsilon_n}{\alpha}\right)^{-2}$$

Expand $\left(1+\frac{\varepsilon_n}{\alpha}\right)^{-2}$ by Binomial theorem

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \cdots \quad \text{(Binomial)}$$

$$\varepsilon_{n+1} = p\alpha - \alpha + p\varepsilon_n + q\alpha\left[1 - 2\frac{\varepsilon_n}{\alpha} + \frac{(-2)(-2-1)}{2!}\frac{\varepsilon_n^2}{\alpha^2} + \cdots\right]$$

$$\varepsilon_{n+1} = p\alpha - \alpha + p\varepsilon_n + q\alpha\left[1 - \frac{2\varepsilon_n}{\alpha} + 3\frac{\varepsilon_n^2}{\alpha^2} + \cdots\right]$$

$$\varepsilon_{n+1} = p\alpha - \alpha + p\varepsilon_n + q\alpha\left[1 - \frac{2\varepsilon_n}{\alpha}\right]$$

$$\varepsilon_{n+1} = p\alpha - \alpha + p\varepsilon_n + q\alpha - \frac{2q\cancel{\alpha}\varepsilon_n}{\cancel{\alpha}}$$

$$\boxed{\varepsilon_{n+1} = p\alpha - \alpha + p\varepsilon_n + q\alpha - 2q\varepsilon_n}$$

$$\varepsilon_{n+1} = \alpha(p - 1 + q) + \varepsilon_n(p - 2q)$$

Comparing on both sides

$$p - 1 + q = 0 \quad — ③$$

$$p - 2q = 0 \quad — ④$$

From Eq^n ③ & ④

$$q = 1/3, \quad p = 2/3$$

Hence, the sol^n is

$$\boxed{p = 2/3, \quad q = 1/3}$$

(1) It is well known fact that

$$\boxed{4x = x + x + x + x}$$

But, when Arithmatic is performed using normalized floating point representation, then above condition may not hold true i.e:

$$\boxed{4x \neq x + x + x + x}$$

Ex→ Let $x = 0.6667E0$

L.H.S → $4x = 4 \times 0.6667E0$

$= .4 \times 10^1 \times 0.6667E0$

$= .4000E1 \times 0.6667E0$

$4x = 0.2667E1$

R.H.S → $x + x + x + x = 0.6667E0 + 0.6667E0 + 0.6667E0 + 0.6667E0$

$= 1.3334E0 + 0.6667E0 + 0.6667E0$

$= 0.1333E1 + 0.6667E0 + 0.6667E0$

$= 0.1333E1 + 0.0666E0 + 0.6667E0$

$= 0.1999E1 + 0.6667E0$

$= 0.1999E1 + 0.0666E0$

$x + x + x + x = 0.2665E1$

$$\boxed{L.H.S. \neq R.H.S.}$$

(2) Associative and distributive law may not hold in floating point representation.

Associative law for addition→

$$x + (y + z) = (x + y) + z$$

Distributive law→

$$x \times (y + z) = (x \times y) + (x \times z)$$

let $x = 0.4000E0$

$y = 0.2000E0$

$z = 0.1000E0$

Associative law→

L.H.S. → $x + (y + z) = 0.4000E0 + (0.2000E0 + 0.1000E0)$

$= 0.4000E0 + (0.3000E0)$

$x + (y + z) = 0.7000E0$

R.H.S. → $(x + y) + z = (0.4000E0 + 0.2000E0) + 0.1000E0$

$= 0.6000 E0 + 0.1000 E0$

$(x+y)+z = 0.7000 E0$

$\boxed{L.H.S. = R.H.S.}$

Q→ $x = 0.4845$, $y = 0.4800$

Calculate the value of $\frac{x^2-y^2}{x+y}$ using FPA. Capare this with the value of $x-y$.

Given $\frac{x^2-y^2}{x+y}$

Nr. part → $x^2-y^2 = (x \times x) - (y \times y)$

$= (0.4845 \times 0.4845) - (0.4800 \times 0.4800)$

$= (.2347) - (0.2304)$

$x^2-y^2 = .0043 = .4300E-2$

Dr. part → $x+y = 0.4845 + 0.4800$

$x+y = 0.9645$

$\frac{x^2-y^2}{x+y} = \frac{.4300E-2}{.9645E0}$

$= .4458E-2$

$x-y = 0.4845 - 0.4800$

$= .0045$

$= .4500E-2$

$\Rightarrow \boxed{\frac{x^2-y^2}{x+y} \neq x-y}$

Q→ Find the sol^n of the following Eq^n using 4 digit mantissa

$x^2 - 1000x + 25 = 0$

Compare the given Eq^n with $ax^2+bx+c=0$ & get the following values

$a = 1 = .1000E1$

$b = -1000 = -.1000E4$

$c = 25 = .2500E2$

Sol^n of given Eq^n is-

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

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$$= \frac{.1000E4 \pm \sqrt{(.1000E4 \times .1000E4) - 4 \times .1000E1 \times .2500E2}}{2 \times .1000E1}$$

$$= \frac{.1000E4 \pm \sqrt{(.1000E4 \times .1000E4) - .4000E1 \times .1000E1 \times .2500E2}}{.2000E1 \times .1000E1}$$

$$= \frac{.1000E4 \pm \sqrt{0.0100E8 - 0.0400E2 \times .2500E2}}{0.0200E2}$$

$$= \frac{.1000E4 \pm \sqrt{.0100E8 - 0.0100E4}}{0.0200E2}$$

$$= \frac{.1000E4 \pm \sqrt{0.0100E8}}{0.0200E2}$$

$$= \frac{.1000E4 \pm 0.1000E4}{0.0200E2}$$

$$= \frac{.2000E4}{0.0200E2}$$

$$= 10E2$$

$$= .0010E4$$